Efficient Self-Consistent Learning of Gate Set Pauli Noise

Senrui Chen (U of Chicago)

QIP 2025

[1] **SC**, Zhihan Zhang, Liang Jiang, Steve Flammia. arXiv: 2410.03906 (2024)
[2] **SC***, Yunchao Liu*, Matthew Otten, Alireza Seif, Bill Fefferman, Liang Jiang. Nat. Comm. 14, 52 (2023)
[3] On-going collaborations with Ed Chen, Alireza Seif, Laurin Fischer et al. (2025)

Setup

Noise remains a major challenge in QIP

nature

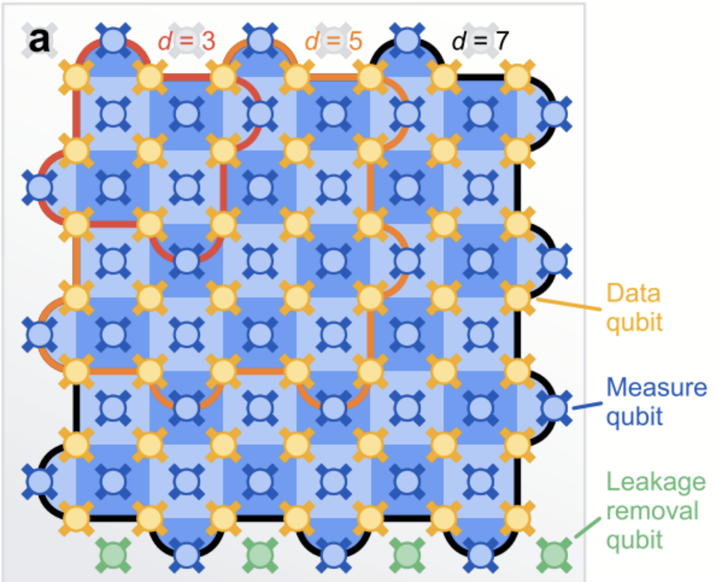
https://doi.org/10.1038/s41586-024-08449-y

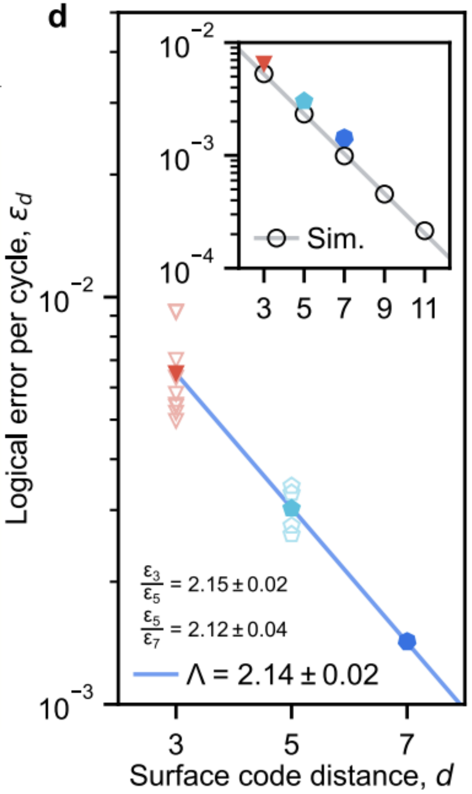
Accelerated Article Preview

Quantum error correction below the surface code threshold

Received: 24 August 2024

Google Quantum AI and Collaborators





Article

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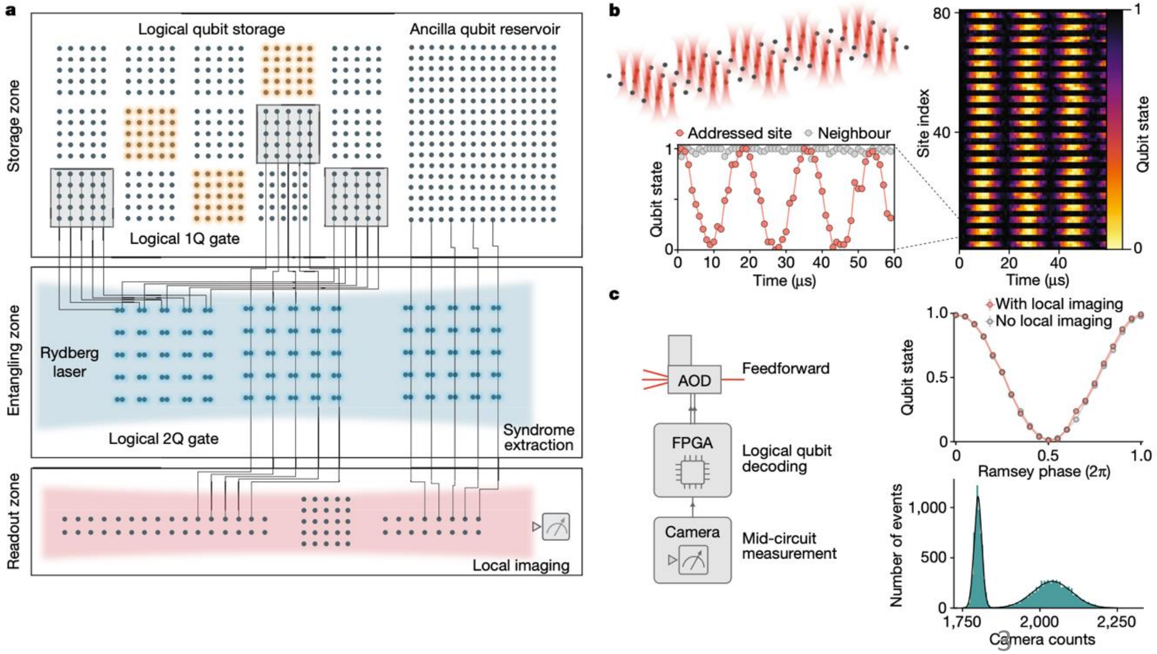
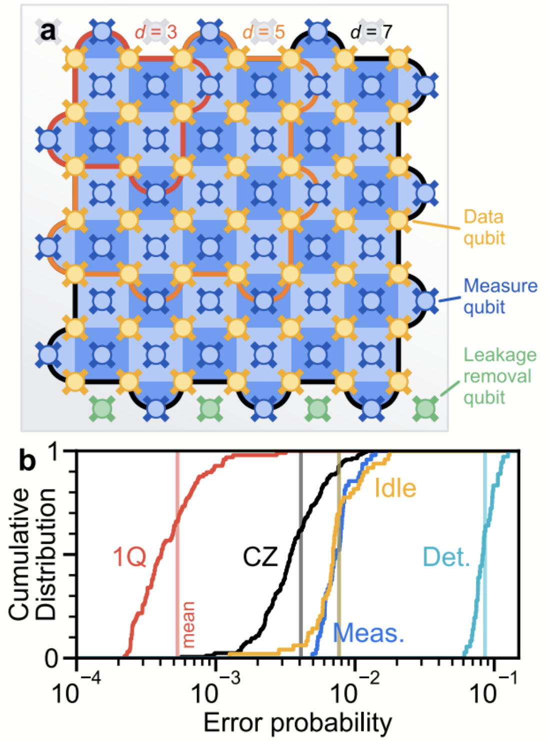


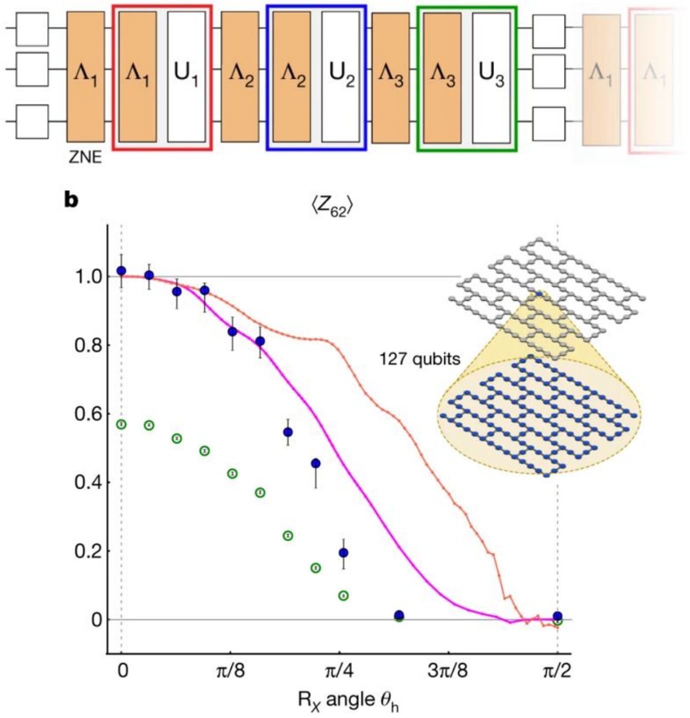
Why noise learning?

Calibration & Benchmarking



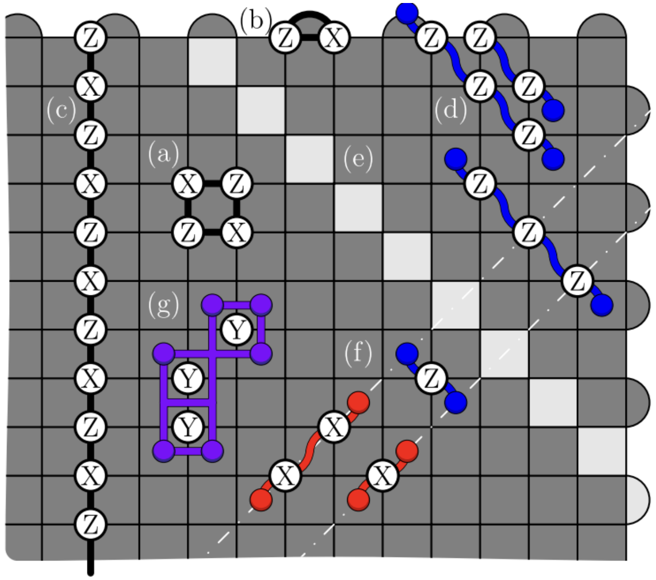
Google Quantum AI, Nature 2024

Quantum Error Mitigation (QEM)



Y Kim et al., IBM Quantum, Nature 2023

Improved designs for Quantum Error Correction (QEC)



J.P. Bonilla Ataides et al., Nat. Commun. 2021

Pauli channels

- n-qubit Pauli channel:

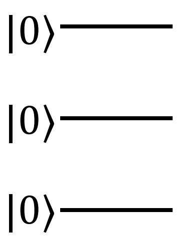
$$\Lambda(\rho) = \sum_{a \in \mathrm{P}^n} p_a P_a \rho P_a = \frac{1}{2^n} \sum_{b \in \mathrm{P}^n} \lambda_b P_b \mathrm{Tr}(P_b \rho)$$

  - $\mathrm{P}^n = \{I, X, Y, Z\}^{\otimes n}$ - n-qubit Pauli group (without phase)
  - $\{p_a\}_a$ – Pauli error rates
  - $\{\lambda_b\}_b$ – Pauli fidelities or Pauli eigenvalues, $\Lambda(P_b) = \lambda_b P_b$

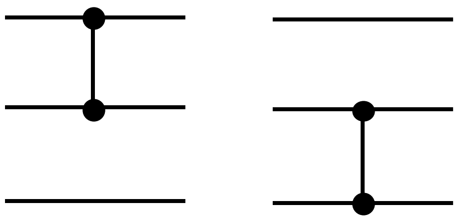
- Symmetric Pauli channel: Pauli fidelities only depend on *Pauli patterns*
  - **Pauli pattern** (i.e., support): $pt(XIZYI) \mapsto 10110$

Pauli noise model

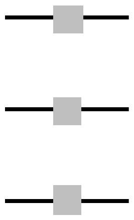
- Consider an n-qubit system with the following **operation set**:



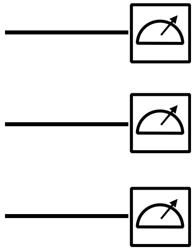
Initialization to $|0\rangle^{\otimes n}$



Layer of multi-qubit Clifford gates
$\mathfrak{G} = \{\mathcal{G}\}$



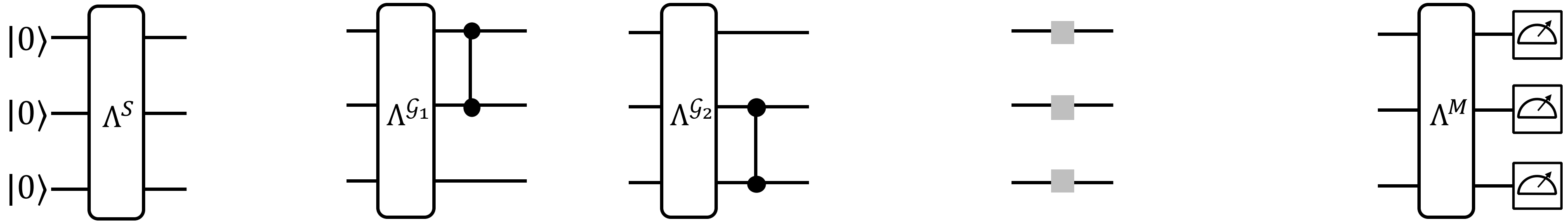
Layer of arbitrary
1-qubit gates $\bigotimes_j \mathcal{U}_j$



Computational-basis
measurement $\{E_j\}_j$

Pauli noise model

- Consider an n-qubit system with the following **operation set**: (Λ -- Pauli channel)

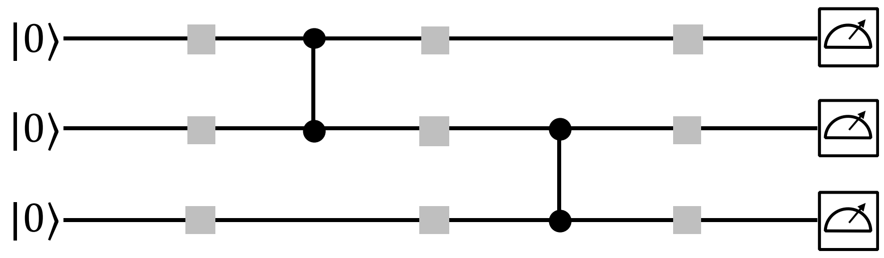


$\tilde{\rho} = \Lambda^S(|0\rangle\langle 0|^{\otimes n})$ $\tilde{\mathcal{G}} = \mathcal{G} \circ \Lambda^{\mathcal{G}}$ $\otimes_j \mathcal{U}_j$ noise negligible $\tilde{E}_j = \Lambda^M(E_j)$

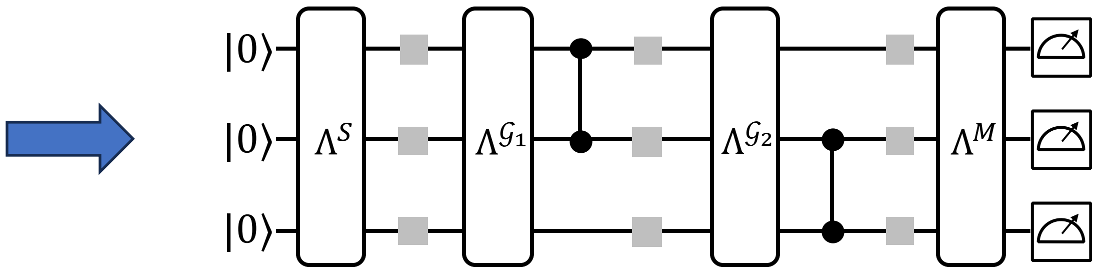
- Λ^S, Λ^M are symmetric Pauli channels. $\Lambda^{\mathcal{G}}$ are $\mathcal{G}$-dependent Pauli channels
- Ensured via Randomized Compiling *[Wallman and Emerson 16]* given good 1q controls
- Applications: Benchmarking *[Erhard et al. 19]*, Quantum Error Mitigation *[Y. Kim et al. Nature 2023]*, Optimized Decoder for QEC *[E Chen et al. PRL. 2022]*, …

Pauli noise model with ansatzes

- **Complete** noise model: Parametrized by full-parameter Pauli channels $\left\{\Lambda^S, \Lambda^M, \{\Lambda^{\mathcal{G}}\}_{\mathcal{G}}\right\}$
  - Contains **exponentially** many parameters. Difficult to learn and use.

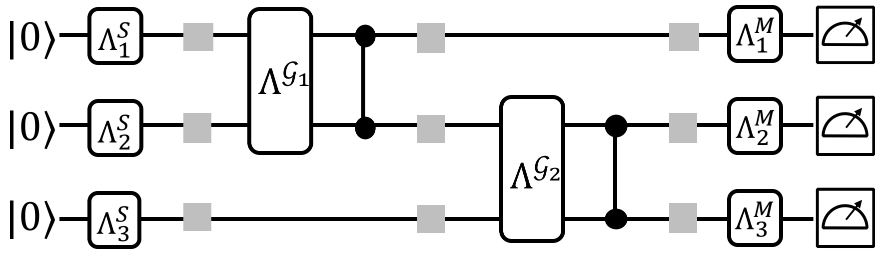


Ideal

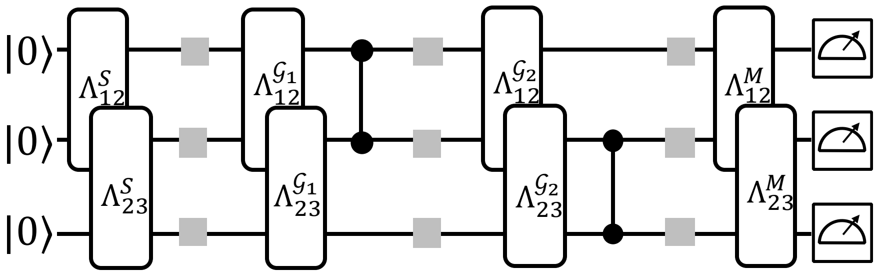


noisy

- **Reduced** noise model: Parametrized by Pauli channels with **efficient** ansatzes

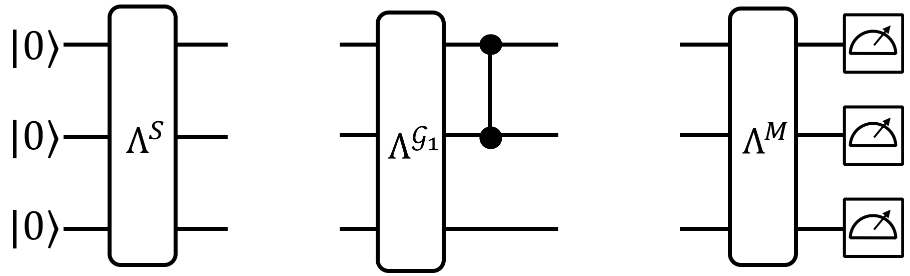


Fully-local



Quasi-local

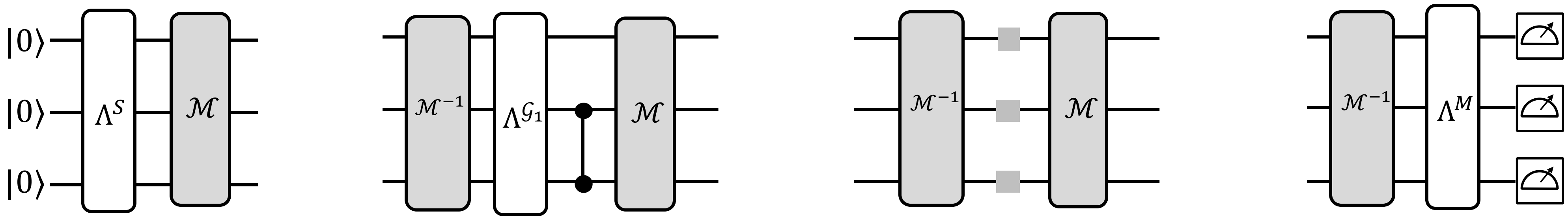
Pauli noise learning



- Main question: How to **self-consistently** learn a Pauli noise model?
  - **Self-consistency**: Using noisy operations to learn about themselves
  - Also known as SPAM-robust learning
- Generically, Λ's cannot be fully-determined self-consistently[1,2].
  - Cycle Benchmarking (CB) *[Erhard et al. 19]*: Learn Λ up to degeneracy
  - ACES *[Flammia 22]*: assumes perfect state preparation
  - RB-tomography *[Kimmel et al. 16]*: requires gate-independent noise
  - Gate Set Tomography *[Nielsen et al. 21]*: gauge freedom exists
- This can be seen using **gauge transformations** as follows

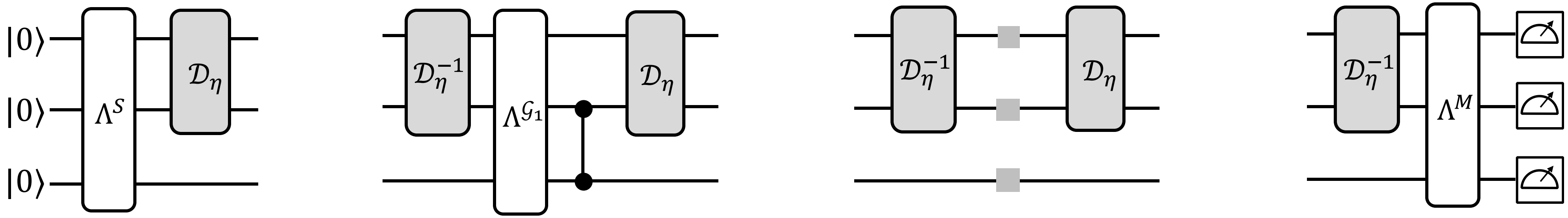
[1] Huang, Flammia, Preskill. (2022). [2] **SC**, Liu, et al. Nat. Commun. (2023).

- Consider the following transformation of model params with an invertible map $\mathcal{M}$



$\tilde{\rho} \mapsto \mathcal{M}(\tilde{\rho})$ $\quad$ $\tilde{\mathcal{G}} \mapsto \mathcal{M} \circ \tilde{\mathcal{G}} \circ \mathcal{M}^{-1}$ $\quad$ $\bigotimes_i \mathcal{U}_i \mapsto \mathcal{M} \circ \bigotimes_i \mathcal{U}_i \circ \mathcal{M}^{-1}$ $\quad$ $\tilde{E}_j \mapsto (\mathcal{M}^{-1})^*(E_j)$

- By choosing $\mathcal{M}$ appropriately, one can preserve the Pauli noise model
  - E.g., Depolarizing channels on any subset of qubits $\mathcal{D}_\eta(\rho) = \eta\, I/d + (1-\eta)\, \rho$



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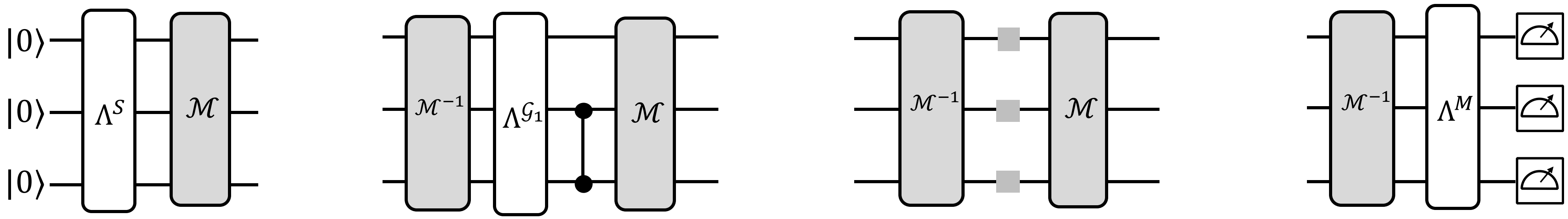

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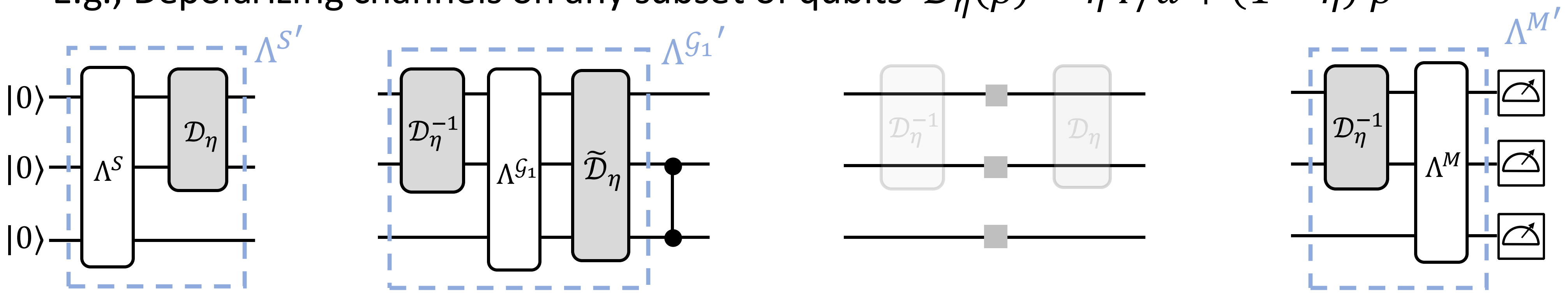
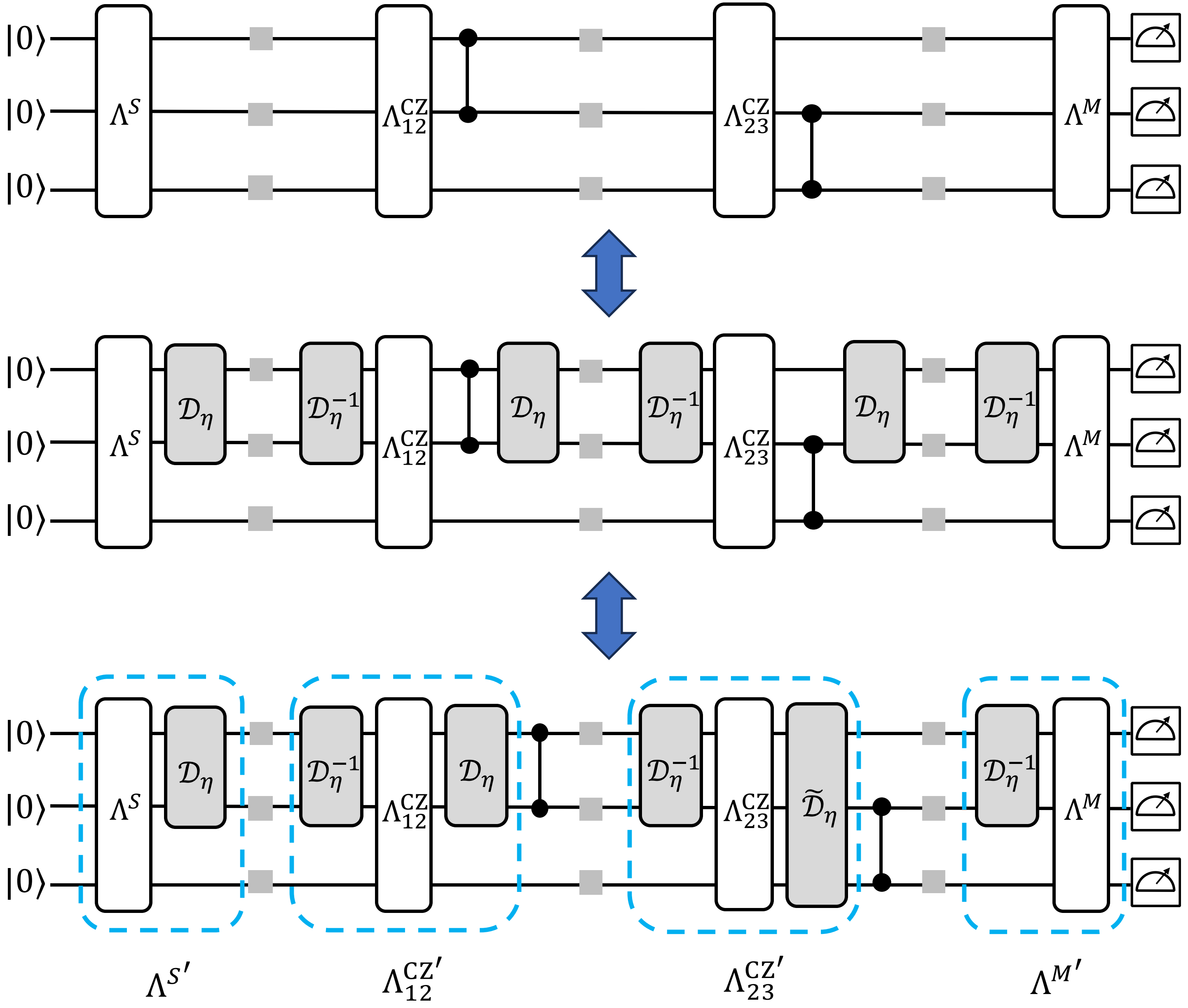
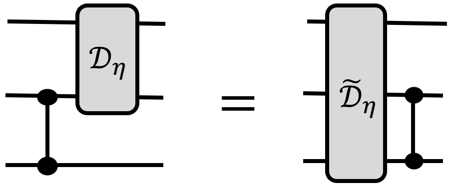
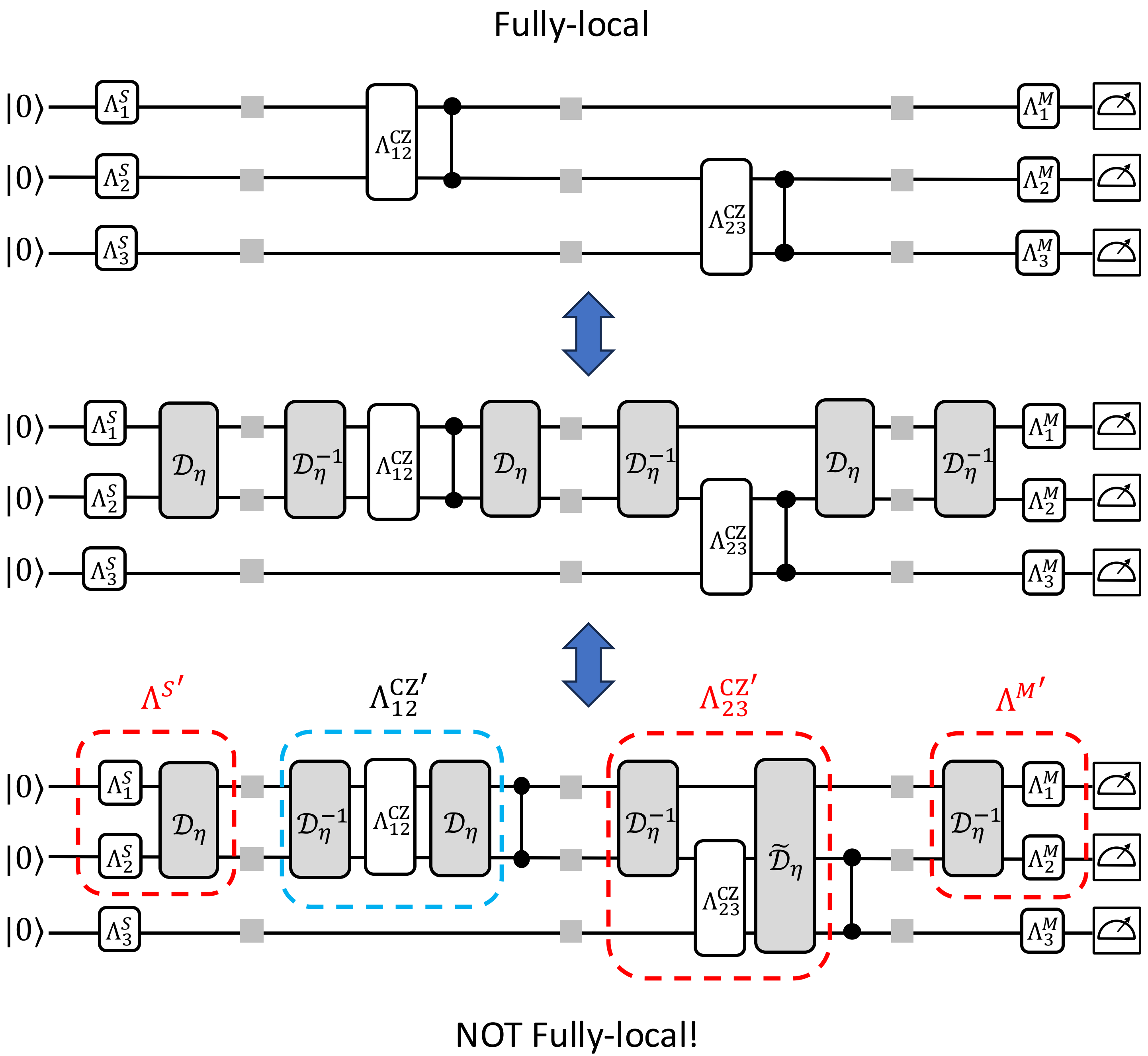


  - This yields a Pauli noise model with different parameters (assuming positivity)
  - We call this a **(subsystem) depolarizing gauge** transformation



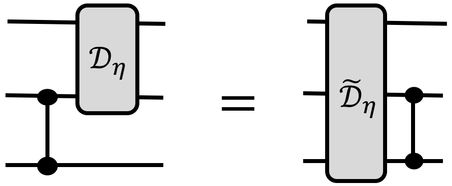
- Two sets of parameters related by a gauge transformation is **indistinguishable** self-consistently.
- If a **function** of noise changes under a gauge transformation, it is **not** self-consistently learnable.
- *Can we characterize all* ***learnable functions*** *and* ***gauges*** *of a Pauli noise model?*





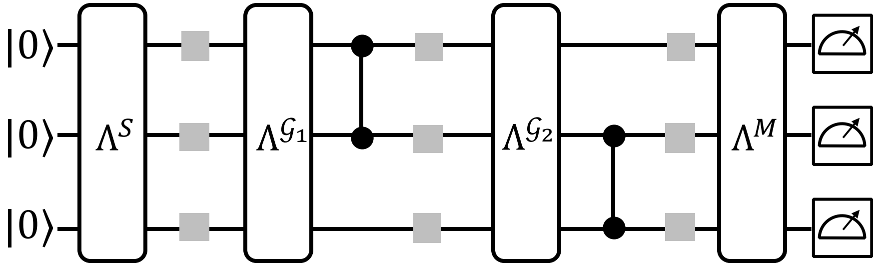
Reduced model

- With additional ansatzes, certain gauge transformations become **invalid.**
- In this example, the transformed $\Lambda^{S'}, \Lambda^{CZ'}_{23}, \Lambda^{M'}$ are not within the fully-local ansatz. Thus, the gauge is invalid.
- *Can we characterize all* ***learnable functions*** *and* ***gauges*** *of a generic reduced Pauli noise model?*



Solutions

Pauli noise in linear space



- Start with the **complete** model

Parameter vector $\boldsymbol{x}$ — $\boldsymbol{x} = [x_a^\star := -\log \lambda_a^\star]^T$ for all Pauli a and all Pauli channels $\star \in \mathfrak{G} \cup \{S, M\}$ ⇒ Parameter space X

Experiments $\boldsymbol{F}_\mathcal{C}\colon X \mapsto \mathbb{R}^{2^n}$ — $\boldsymbol{F}_\mathcal{C}[j] = \mathrm{Tr}(\tilde{E}_j \tilde{\mathcal{C}}(\tilde{\rho}_0))$ for a gate sequence $\mathcal{C}$

Learnable functions $\boldsymbol{f} \in X^*$ — $\boldsymbol{f}(\boldsymbol{x}) = \boldsymbol{f} \cdot \boldsymbol{x}$ that can be determined by some set of experiments $\{\boldsymbol{F}_\mathcal{C}\}$ ⇒ Learnable space L

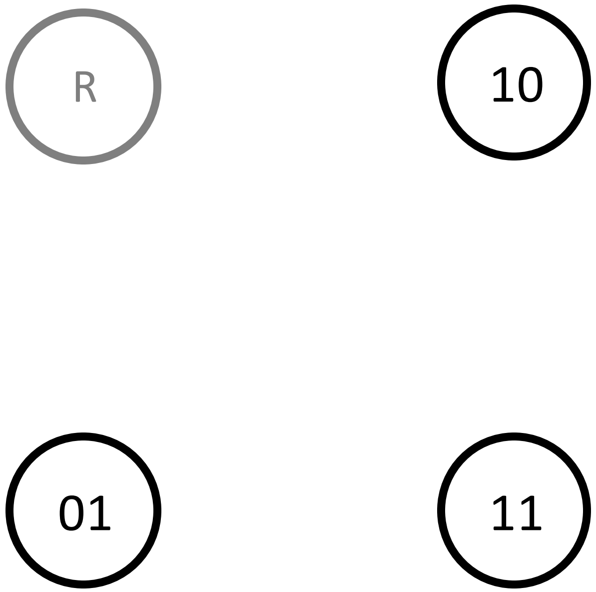
Gauge vectors $\mathfrak{d} \in X$ — $\boldsymbol{F}_\mathcal{C}(\boldsymbol{x}) = \boldsymbol{F}_\mathcal{C}(\boldsymbol{x} + \mathfrak{d})$ for any $\boldsymbol{F}$ and $\boldsymbol{x} \in X$ ⇒ Gauge space T

- How to characterize **learnable space** L and **gauge space** T?

Graph theory to the rescue

- Define Pauli **pattern transfer graph** (PTG)[1]
  - $2^n - 1$ Pauli pattern nodes and 1 Root node

Pattern: $\mathrm{pt}(XIZYI) \mapsto 10110$



$\mathfrak{G} \coloneqq \{\mathcal{G} = \mathrm{CZ}\}$

Modified from [**SC**, Liu, et al. “The learnability of Pauli noise”. Nat. Commun. (2023).]

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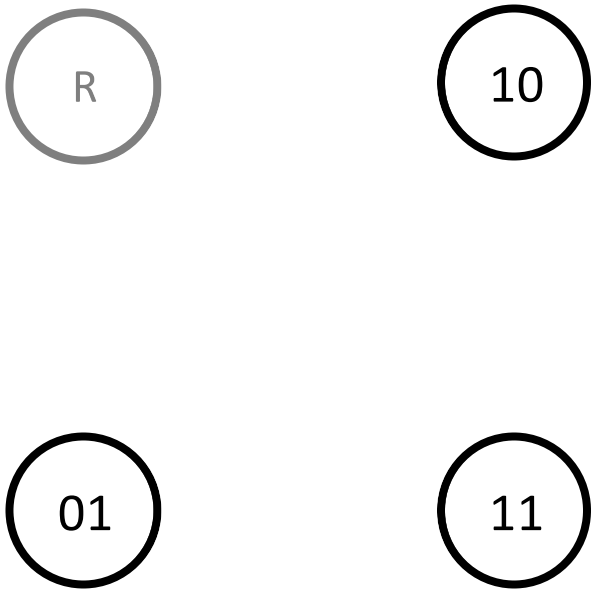

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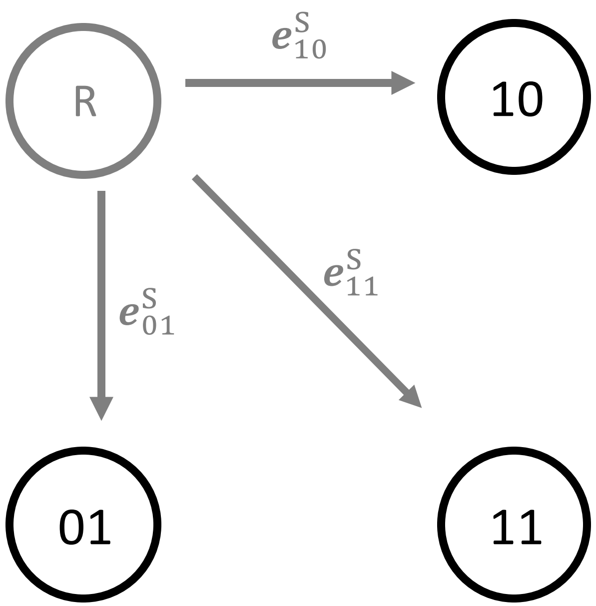

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    - M fidelities $\boldsymbol{e}_t^M$: From pattern t to Root

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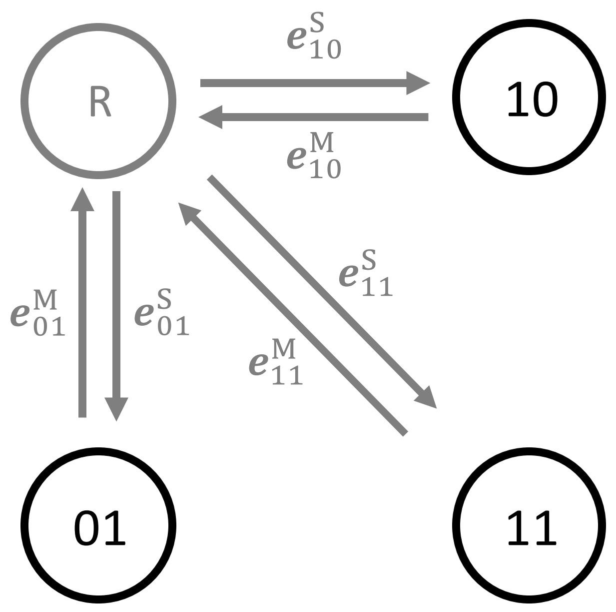

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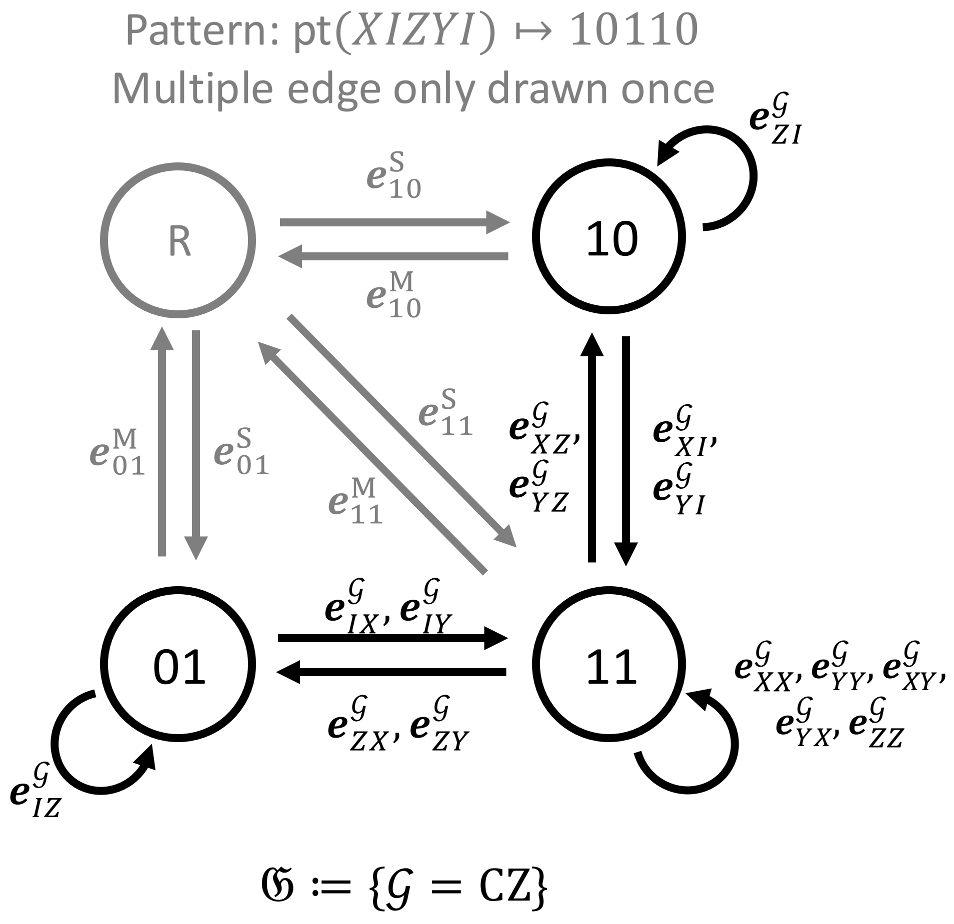
    - Gate fidelities $\boldsymbol{e}_a^{\mathcal{G}}$: From pattern($a$) to pattern($\mathcal{G}(a)$)



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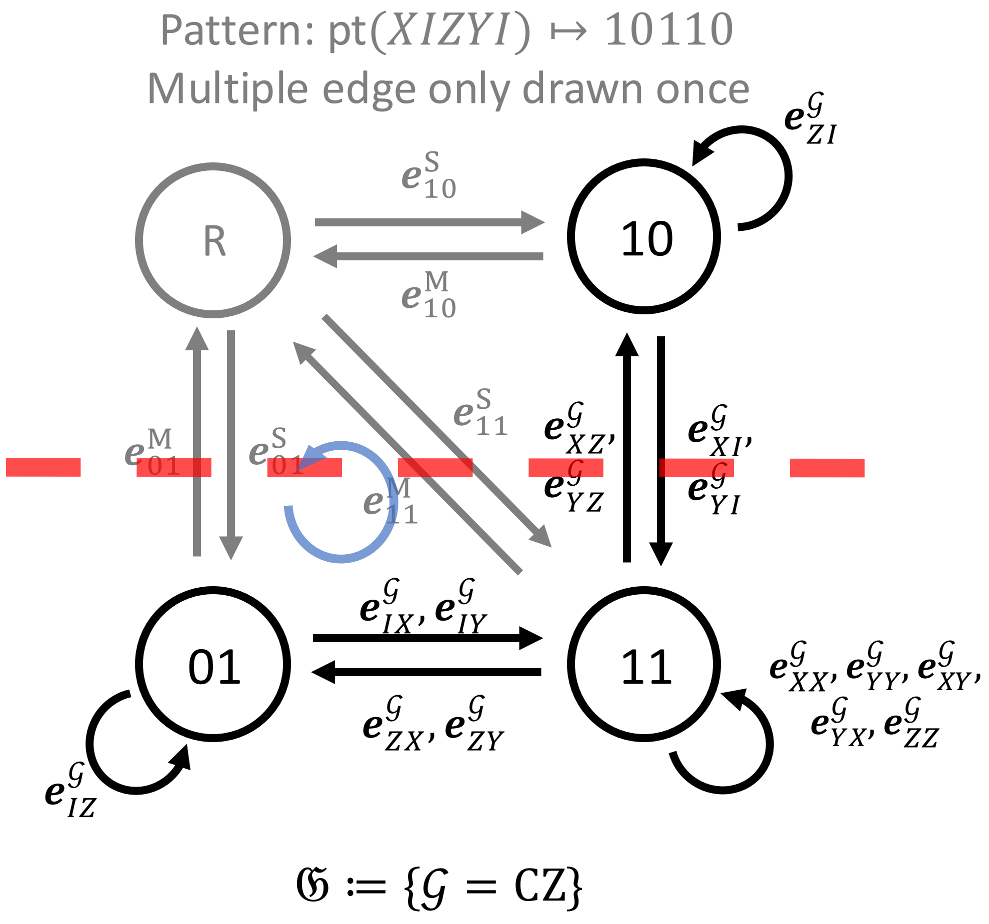
    - M fidelities $\boldsymbol{e}_t^M$: From pattern t to Root
    - Gate fidelities $\boldsymbol{e}_a^{\mathcal{G}}$: From pattern$(a)$ to pattern$(\mathcal{G}(a))$
- Linear spaces on graph:
  - **Edge space** E: spanned by all edges $\{\boldsymbol{e_i}\}$
  - **Cycle space** Z: spanned by all cycle vectors
    - e.g. $\boldsymbol{e}_{01}^S + \boldsymbol{e}_{IX}^{\mathcal{G}} + \boldsymbol{e}_{11}^M$
  - **Cut space** U: spanned by all cut vectors
    - e.g. $\boldsymbol{e}_{01}^S + \boldsymbol{e}_{11}^S + \boldsymbol{e}_{XI}^{\mathcal{G}} + \boldsymbol{e}_{XI}^{\mathcal{G}} - \boldsymbol{e}_{01}^M - \boldsymbol{e}_{11}^M - \boldsymbol{e}_{XZ}^{\mathcal{G}} - \boldsymbol{e}_{YZ}^{\mathcal{G}}$
- Lemma. $E = Z \oplus^{\perp} U$ $\oplus^{\perp}$: orthogonal complement



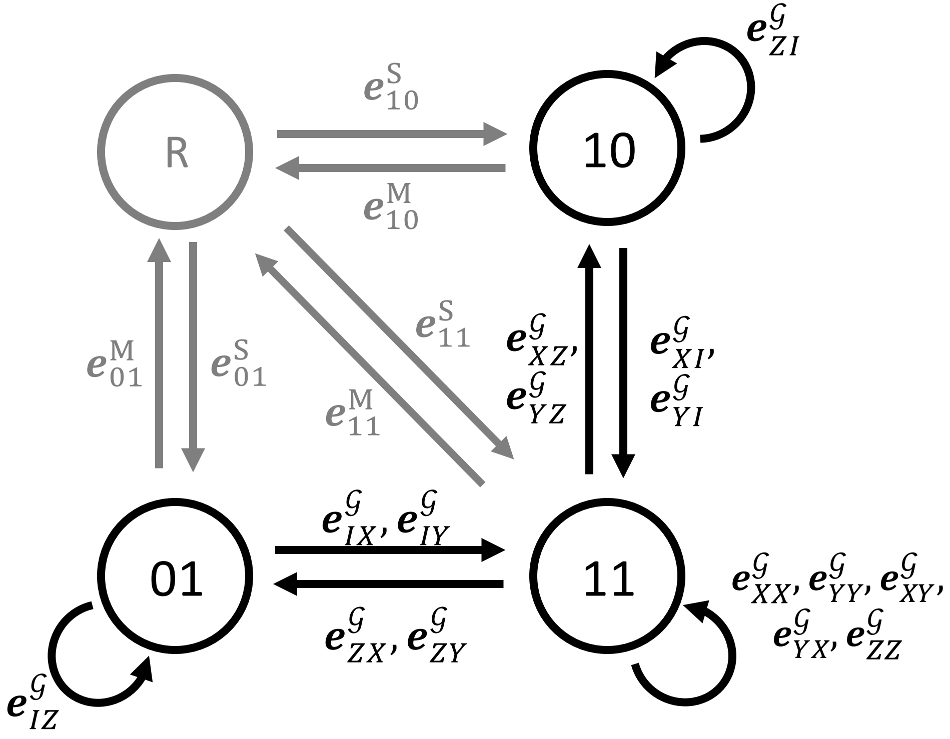
Modified from [**SC**, Liu, et al. "The learnability of Pauli noise". Nat. Commun. (2023).]

Learnability of complete Pauli noise

- Theorem (informal):

E	$=$	Z	$\oplus^{\perp}$	U
(edge)		(cycle)		(cut)
$\parallel$		$\parallel$		$\parallel$
X	$=$	L	$\oplus^{\perp}$	T
(parameter)		(learnable)		(gauge)

Note: $\oplus^{\perp}$ stands for orthogonal complement



- Proof sketch: show that “cycles are learnable” and “cuts are gauge”

Modified from [**SC**, Liu, et al. “The learnability of Pauli noise”. Nat. Commun. (2023).]

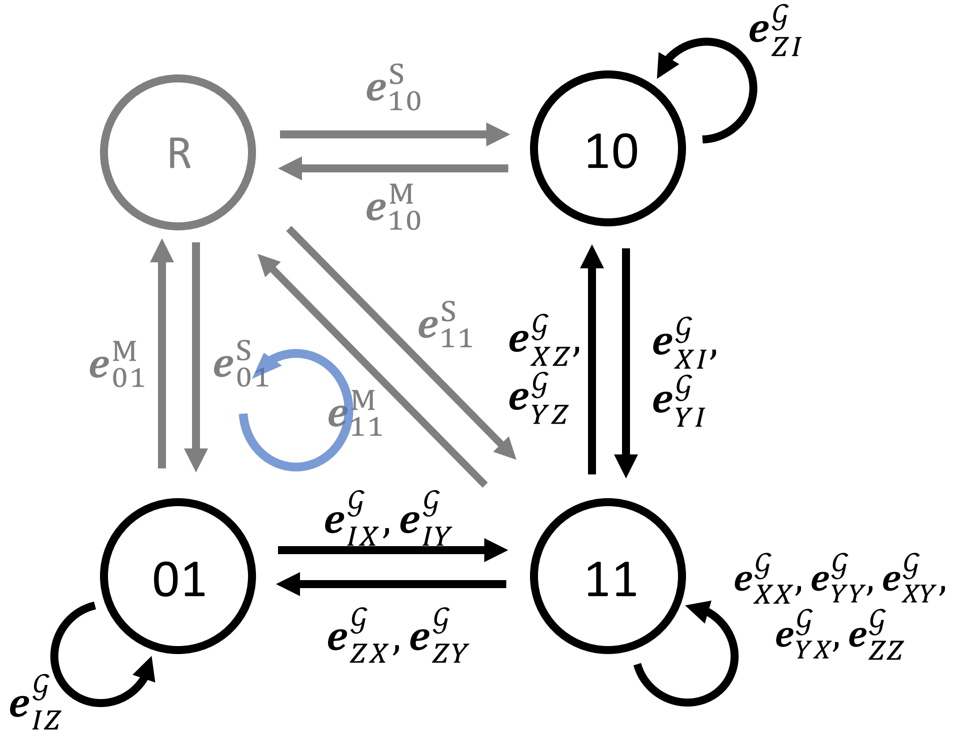
Proof: Cycles are learnable

$$\underset{\text{(edge)}}{E} = \underset{\text{(cycle)}}{Z} \oplus^{\perp} \underset{\text{(cut)}}{U}$$

$$\parallel \qquad \parallel \qquad \parallel$$

$$\underset{\text{(parameter)}}{X} = \underset{\text{(learnable)}}{L} \oplus^{\perp} \underset{\text{(gauge)}}{T}$$

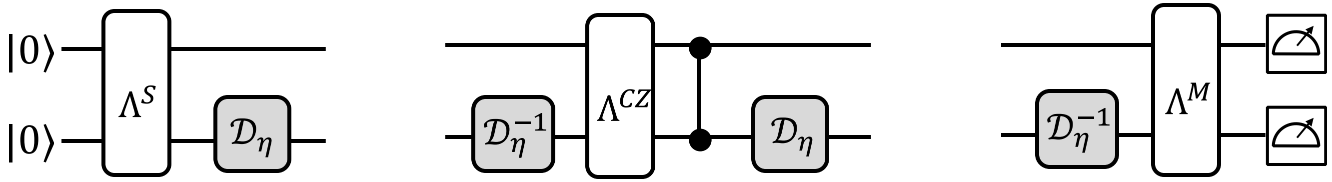
- **Rooted cycle**: cycle passing Root exactly once
- Any **rooted cycle** can be learned by an **experiment**
  - ∃ Clifford gates sequence + Pauli measurements:
  - $\mathrm{Tr}(\tilde{P}\tilde{\mathcal{C}}(\tilde{\rho}_0)) = \lambda^{S}_{\mathrm{pt}(a_1)}\lambda^{\mathcal{G}_1}_{a_1}\lambda^{\mathcal{G}_2}_{a_2}\cdots\lambda^{\mathcal{G}_M}_{a_M}\lambda^{M}_{\mathrm{pt}(a_M)}$
    $= \exp\left(-\left(x^{S}_{\mathrm{pt}(a_1)} + \cdots + x^{M}_{\mathrm{pt}(a_M)}\right)\right)$
    Rooted cycle
  - Can be understood as a **Pauli path**.
  - Thus, rooted cycles are learnable
- By construction, rooted cycles span cycle space Z
  - As Root strongly connects to all other nodes
- Thus, any function in Z is learnable
  - Naive learning algorithm: find a **rooted cycle basis**, learn them one-by-one by running the corresponding Clifford circuits.



Modified from [**SC**, Liu, et al. "The learnability of Pauli noise". Nat. Commun. (2023).]

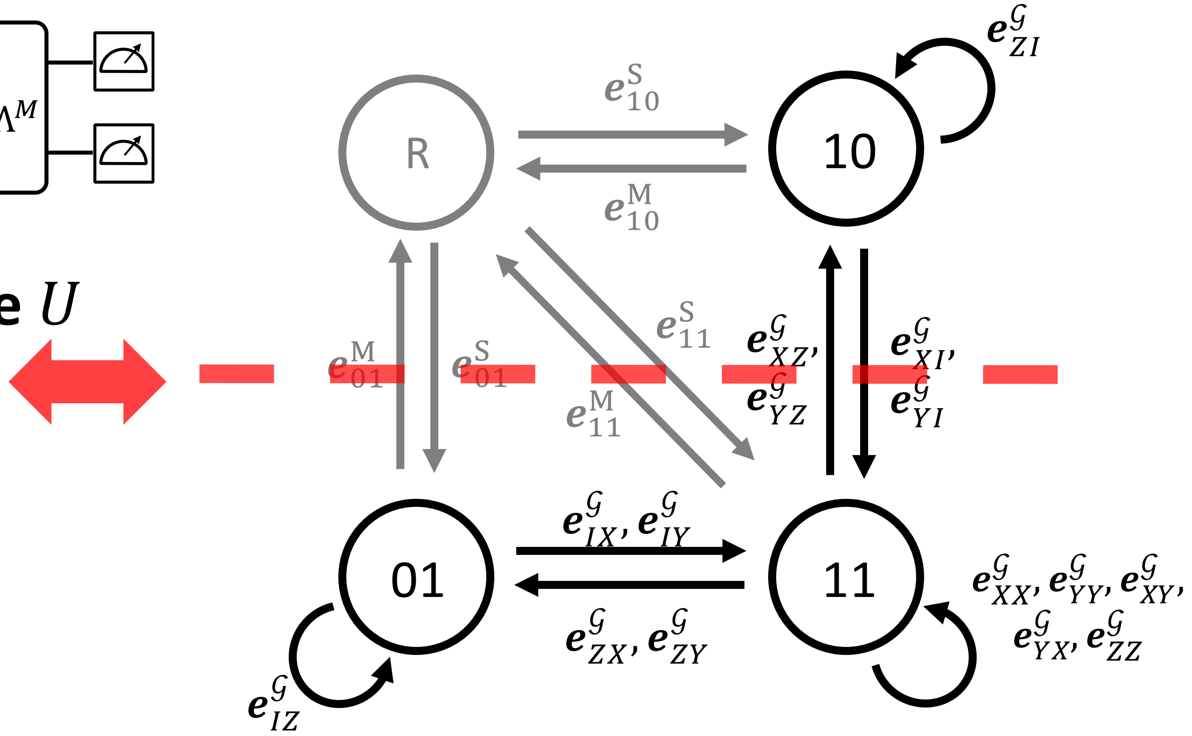
Proof: Cuts are gauges

- Consider the family of **depolarizing gauges**
  - Example: $\mathfrak{d}_{\{2\}}$ - 1q depolarizing gauge on qubit 2



- Those gauges form a basis for the **cut space** U
  - Example of $\mathfrak{d}_{\{2\}}$: prop to {R, 10} / {01, 11}
- Thus, any vector from U is a gauge
  - Corollary: There are always $2^n - 1$ gauge DOFs
  - Subsystem depolarizing gauges gives a basis.

$$E \text{ (edge)} = Z \text{ (cycle)} \oplus^\perp U \text{ (cut)}$$
$$\| \qquad \| \qquad \|$$
$$X \text{ (parameter)} = L \text{ (learnable)} \oplus^\perp T \text{ (gauge)}$$



Modified from [**SC**, Liu, et al. "The learnability of Pauli noise". Nat. Commun. (2023).]

Extension to reduced models

- Define a reduced noise model by $(X_R, \mathcal{Q})$:

  X_R – Reduced parameter space

  $\boldsymbol{r} \in X_R$ – Vector of reduced params.

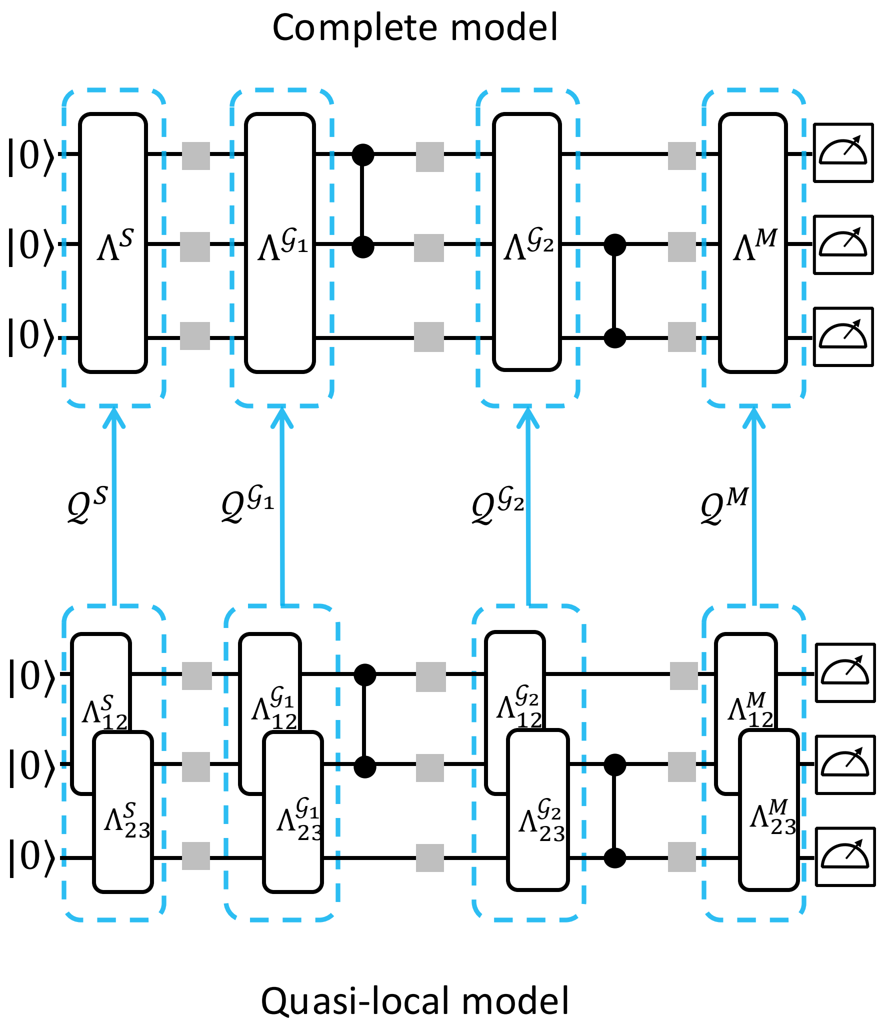
  $\mathcal{Q}: X_R \mapsto X$ – Embedding map onto complete model

  (We require $\mathcal{Q}$ to be **linear** and **injective**)

- Definitions of learnability is similar:
  - Reduced exp. $\boldsymbol{F}_R$: $X_R \mapsto \mathbb{R}^{2^n}$, $\quad \boldsymbol{F}_R(\boldsymbol{r}) \coloneqq \boldsymbol{F}\,(\mathcal{Q}(\boldsymbol{r}))$
  - Reduce learnable space L_R:

    $\{\boldsymbol{f} \in X_R'$ whose values can be determined from some $\{\boldsymbol{F}_R\}$
  - Reduced gauge space T_R:

    $\{\mathfrak{d} \in X_R$ s.t. $\boldsymbol{F}_R(\boldsymbol{r}) = \boldsymbol{F}_R(\boldsymbol{r} + \mathfrak{d})$ for all $\boldsymbol{r} \in X_R$ and $\boldsymbol{F}_R\}$



- How to characterize **reduced learnable space** L_R and **reduced gauge space** T_R?

Learnability of reduced model

- Theorem (informal).

$$
\begin{array}{ccccccc}
E & = & Z & \oplus^{\perp} & U \\
\text{(edge)} & & \text{(cycle)} & & \text{(cut)} \\
\parallel & & \parallel & & \parallel \\
X & = & L & \oplus^{\perp} & T \\
\text{(parameter)} & & \text{(learnable)} & & \text{(gauge)} \\
\uparrow \mathcal{Q} & & \downarrow \mathcal{Q}^{\mathrm{T}} & & \downarrow \mathcal{Q}^{-1} \\
X_R & = & L_R & \oplus^{\perp} & T_R \\
\text{(reduced param)} & & \text{(reduced learnable)} & & \text{(Reduced gauge)}
\end{array}
$$

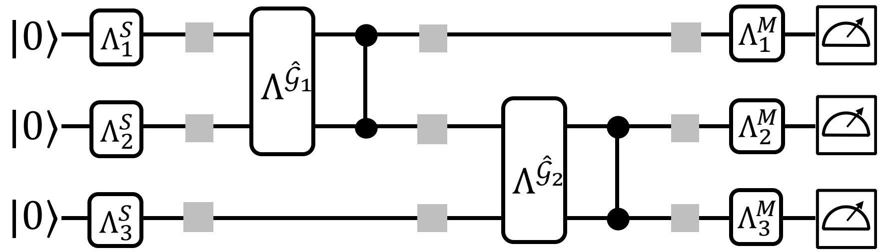
- More precisely:
- $L_R = \mathcal{Q}^{\mathrm{T}}(L) \equiv \{f(\mathcal{Q}(\cdot)) \mid \forall f \in L\}$
  - i.e., L_R is L projected by $\mathcal{Q}^{\mathrm{T}}$
  - $\mathcal{Q}^{\mathrm{T}}$ denotes the conjugate map of $\mathcal{Q}$
- $T_R = \mathcal{Q}^{-1}(T) \equiv \{\boldsymbol{\mathfrak{d}} \in X_R \mid \mathcal{Q}(\boldsymbol{\mathfrak{d}}) \in T\}$
  - i.e., T_R is the preimage of T via $\mathcal{Q}$
  - Equivalently, $\mathcal{Q}(T_R) = T \cap \mathrm{Im}\mathcal{Q}$
    - only (complete) gauge in the image of $\mathcal{Q}$ is valid

- Results: Standard linear algebraic alg. to determine L_R and T_R from graph linear space
- Caveat: Pattern transform graph is **exponential** size! Finding basis for L_R is inefficient
- Remedy: We can analytically characterize L_R and T_R for some concrete cases

[**SC**, Zhihan Zhang, Liang Jiang, Steve Flammia. 2024]

Case study I: Fully-local model

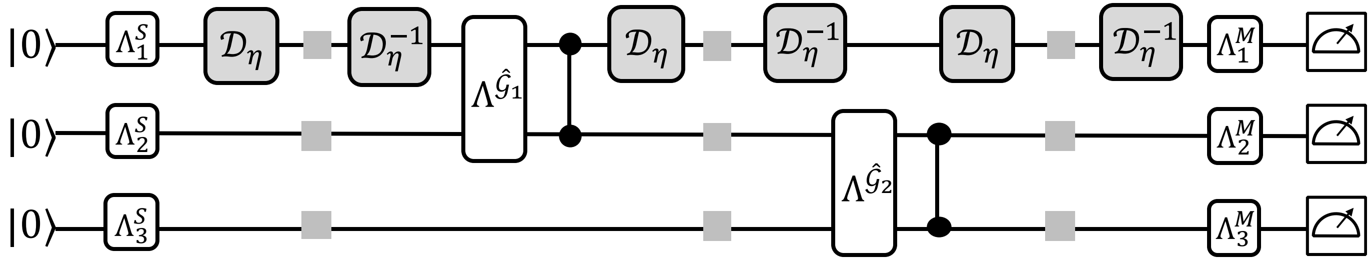
- Fully-local model (aka crosstalk-free):
  - Local SPAM noise: Product of 1q Pauli channel
  - Local Gate noise: channel within gate's support



- Theorem (Learnability of fully-local noise):
  - The embedded gauge space is spanned by **single-qubit depolarizing gauges**, i.e.,

    $\mathcal{Q}(T_R) = \{\, \mathfrak{d}_v : |v| = 1\}$

Example of an 1q depolarizing gauge

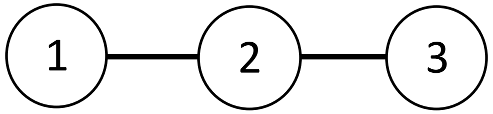


- Remark:
  - Can be generalize to only SPAM or Gate has local noise
  - Efficient design of learning experiments discussed in paper

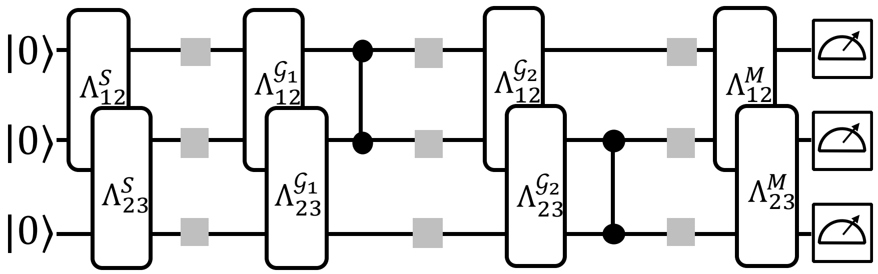
[**SC**, Zhihan Zhang, Liang Jiang, Steve Flammia. 2024]

Case study II: Quasi-local model

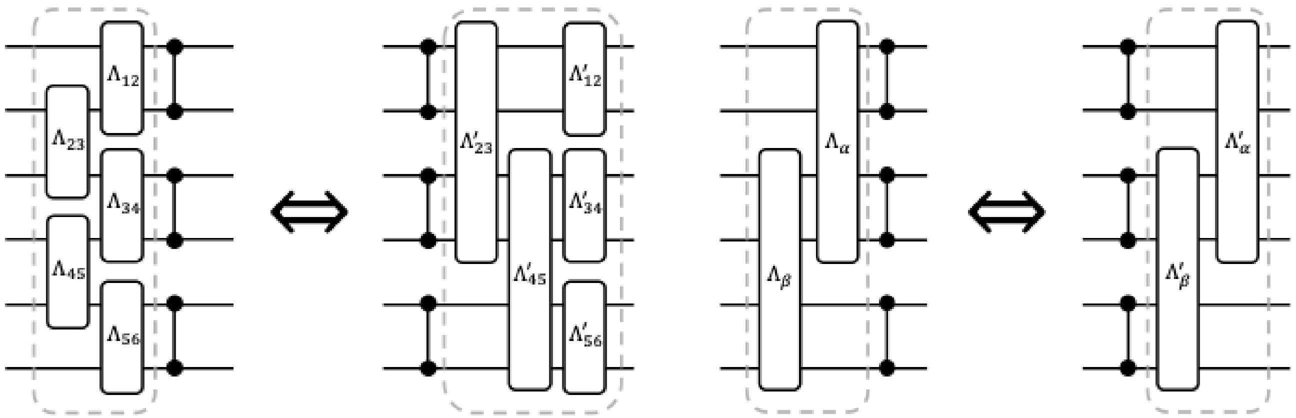
- Quasi-local noise model:
  - Let Ω be the set of all cliques on an n-node graph
  - **Ω-local Pauli channel:** compositions of (possibly negative) Pauli channels on sets of Ω [1-2]
  - Ω-local noise model: All Pauli channels are Ω-local
- Theorem (Learnability of Ω-local noise):
  - Given the noise model is Ω-**covariant**, the embedded gauge space is spanned by **depolarizing gauges** supported on Ω, i.e., $\mathcal{Q}(T_R) = \{ \mathfrak{d}_v : v \in \Omega\}$
- **Covariance** means Ω-locality is preserved when commuting through gates.
- Some common models are not covariant, need to analyze case-by-case



$\Omega = \{\{1\},\{2\},\{3\},\{1,2\},\{2,3\}\}$



Not covariant, unfortunately…



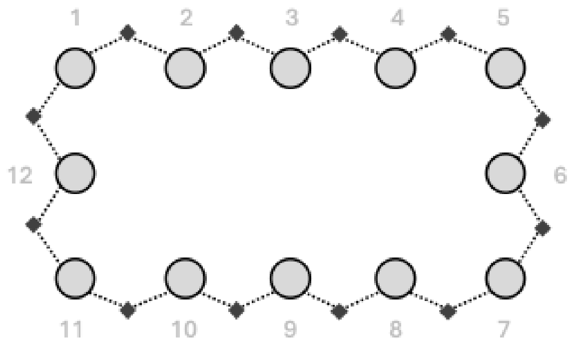
(a) Not covariant

(b) Covariant

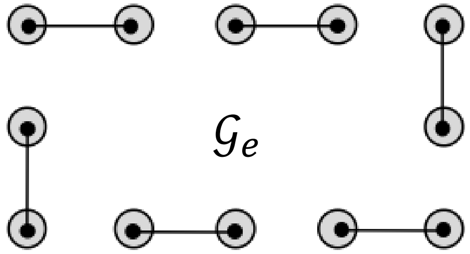
[1] Ewout et al. Nat. Phys. 2023. [2] Wagner et al. PRL 2024.

Case study III: 2-local noise of parallel CZ gates

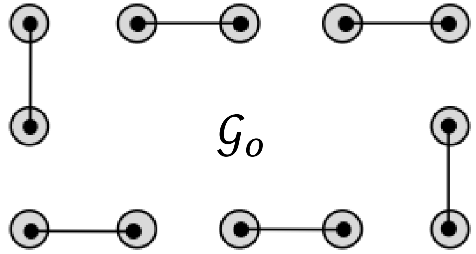
- We analyzed a nearest-neighbor 2-local model from [Ewout et al. Nat. Phys. 2023]
  - Two layers of parallel CZ gates on a 1D ring
  - Gates and SPAM Pauli noise assumed to be 2-local, not covariant!
- We can explicitly compute L_R, T_R in this case
  - Specifically, we show **1-qubit depolarizing gauges** spans $\mathcal{Q}(T_R)$
- One can efficiently and self-consistently learn this noise model
  - Efficient Gauge-consistent error mitigation without "symmetry assumptions"



(a)



(b)

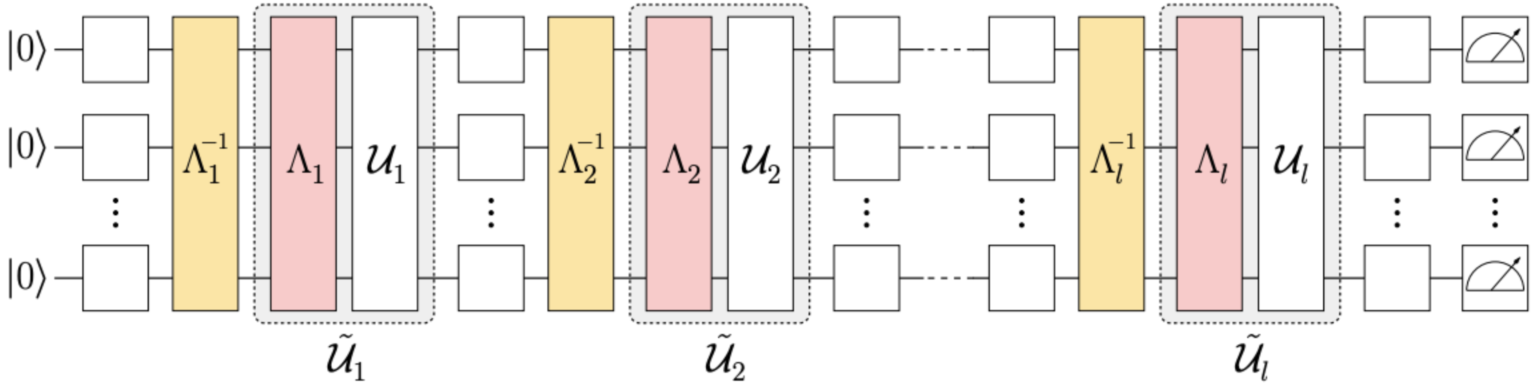


(c)

Applications

Applications in Quantum Error Mitigation

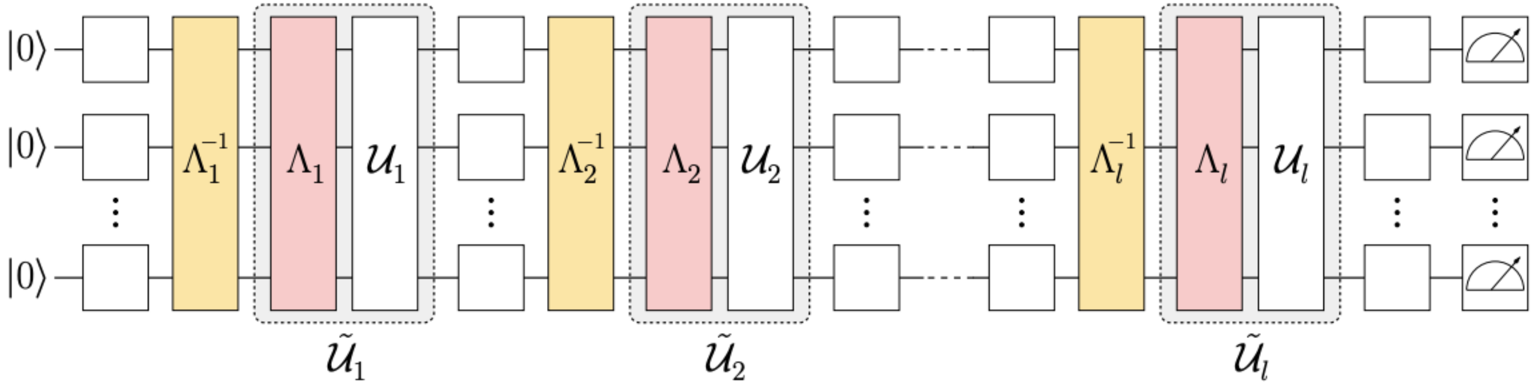
- Pauli noise model has been applied in quantum error mitigation



[Berg et al. IBM Quantum, Nature Physics 2023]

- Example: Probabilistic Error Cancellation (PEC):
- First learn $\Lambda^{\mathcal{G}}$, then implement $\left(\Lambda^{\mathcal{G}}\right)^{-1}$ using quasi-probability sampling.
  - $\Lambda^{-1}(\rho) = \sum_a p_a P_a \rho P_a$ where p_a can be negative.
  - Sample from $|p_a| / \sum_a |p_a|$, add negative sign in postprocessing
  - Overhead exponentially depends on $\gamma \coloneqq \sum_a |p_a|$
- Similar procedure for M noise mitigation, Ignore SP noise.

Applications in Quantum Error Mitigation

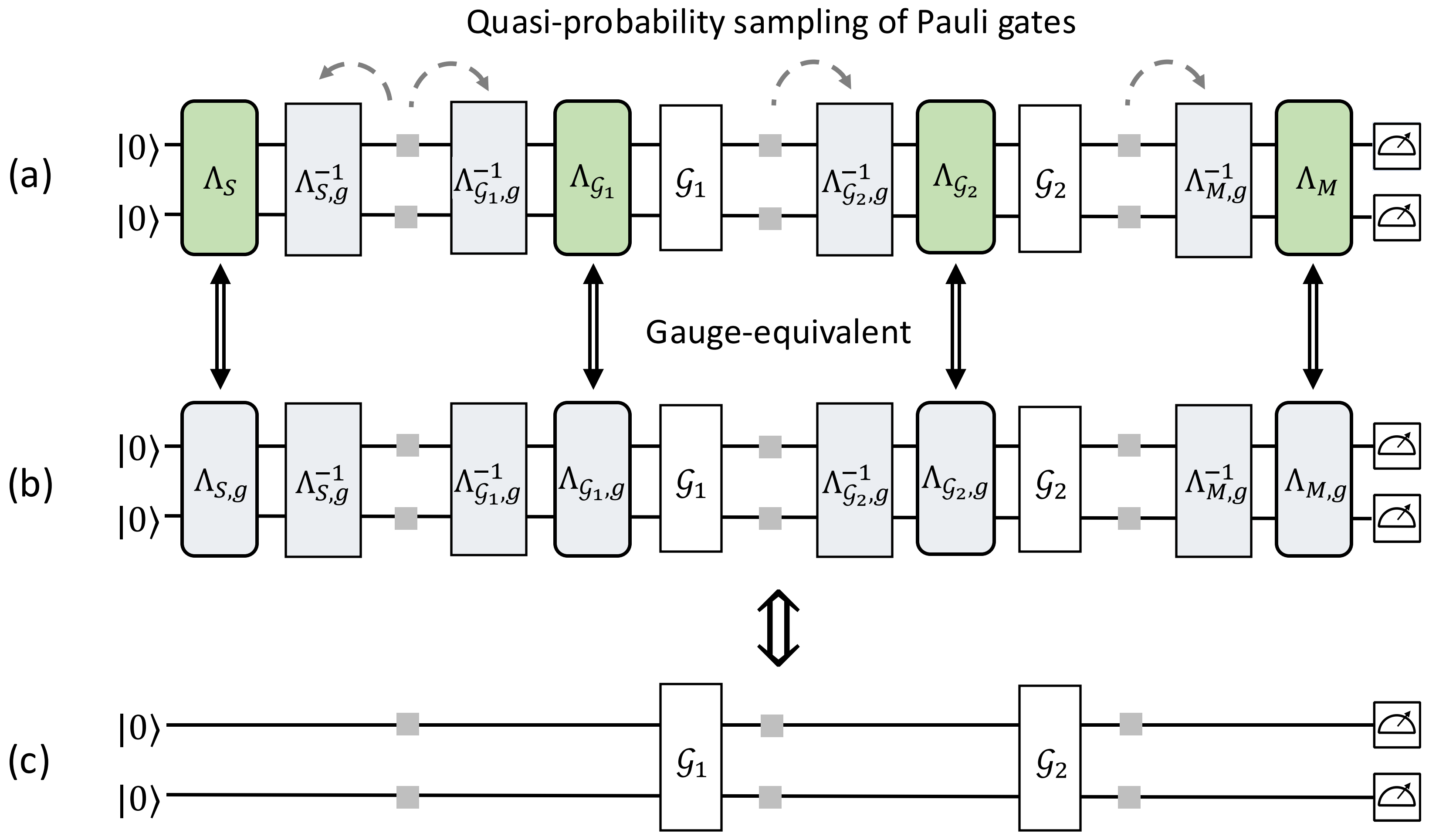


[Berg et al. IBM Quantum, Nature Physics 2023]

- **Caveat**: Due to learnability issue, $\Lambda^{\mathcal{G}}$ cannot be fully SPAM-robustly learned.
- Existing paper resort to an “symmetry assumptions” which is not physically justified.
  - E.g.: Breaking degeneracy of $\lambda_{XI}^{\mathrm{CNOT}}$ and $\lambda_{XX}^{\mathrm{CNOT}}$ by assuming they are equal
- We show such assumptions are not necessary
- **Scalable Gauge-consistent QEM** based on gate-set Pauli noise learning.

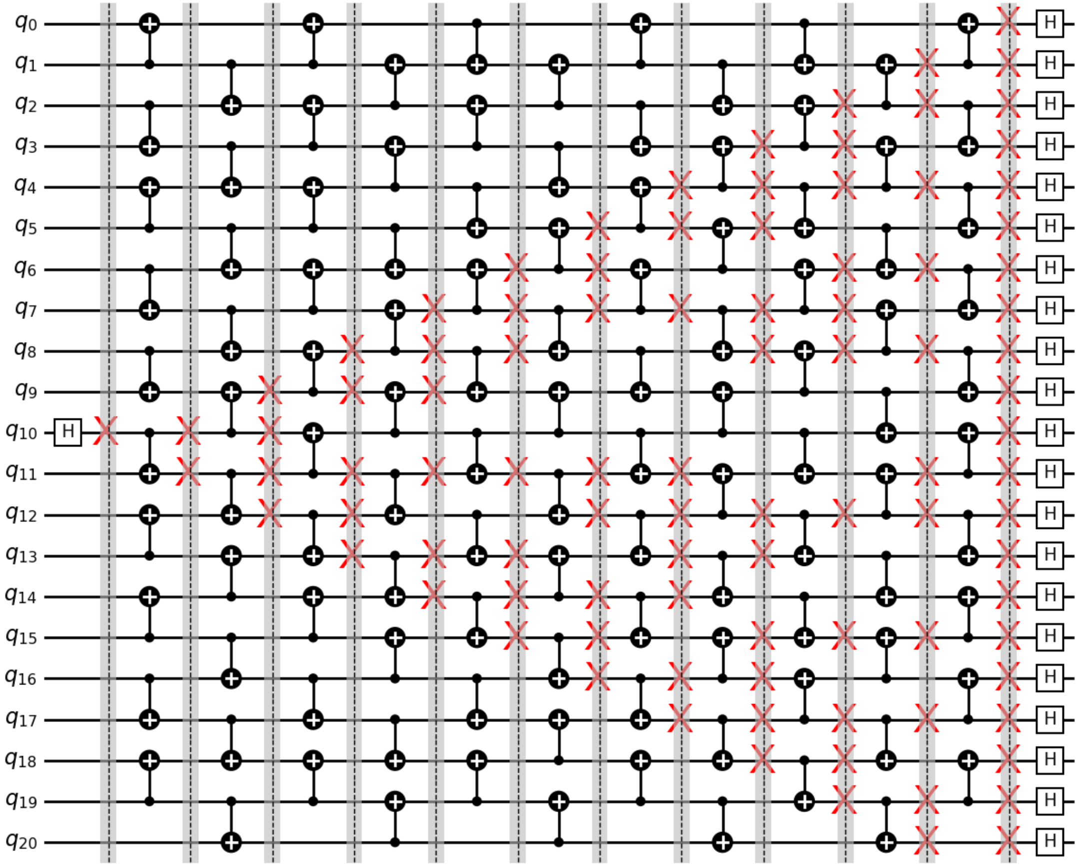
[Berg et al. IBM Quantum, Nature Physics 2023]

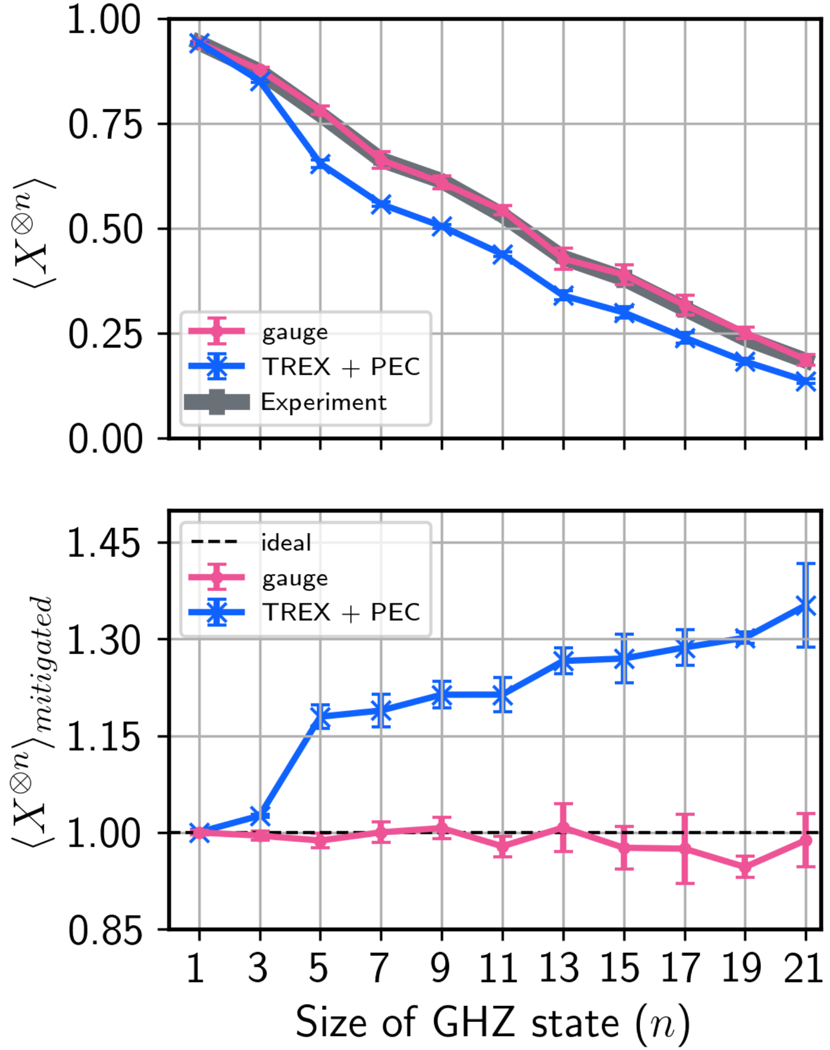
PEC with gauge consistency



- Generalized to reduced Pauli noise model with efficient ansatzes

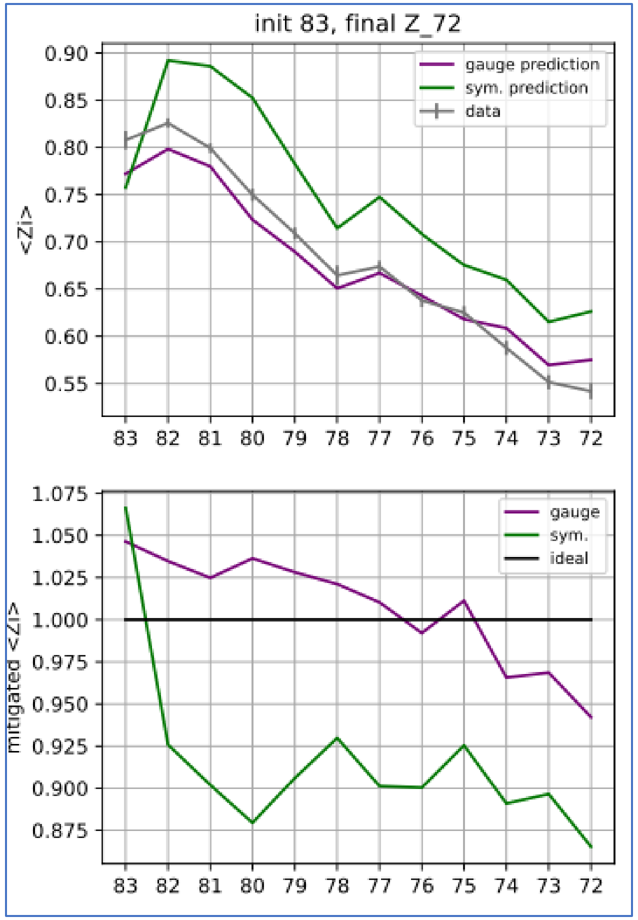
21Q dense GHZ:
Error mitigation workflow

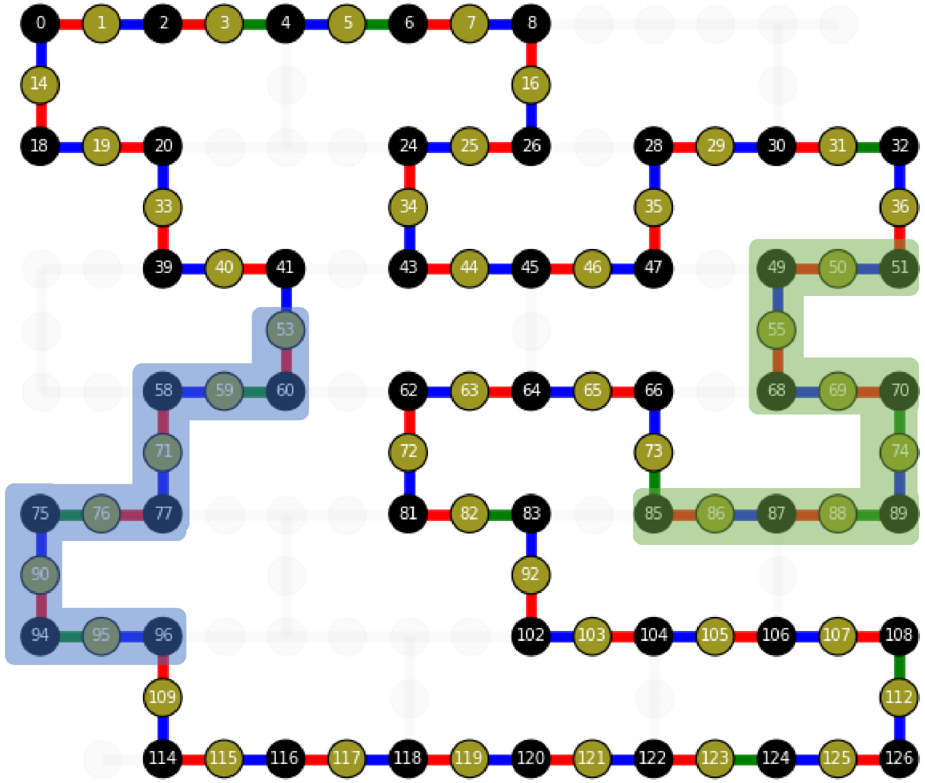


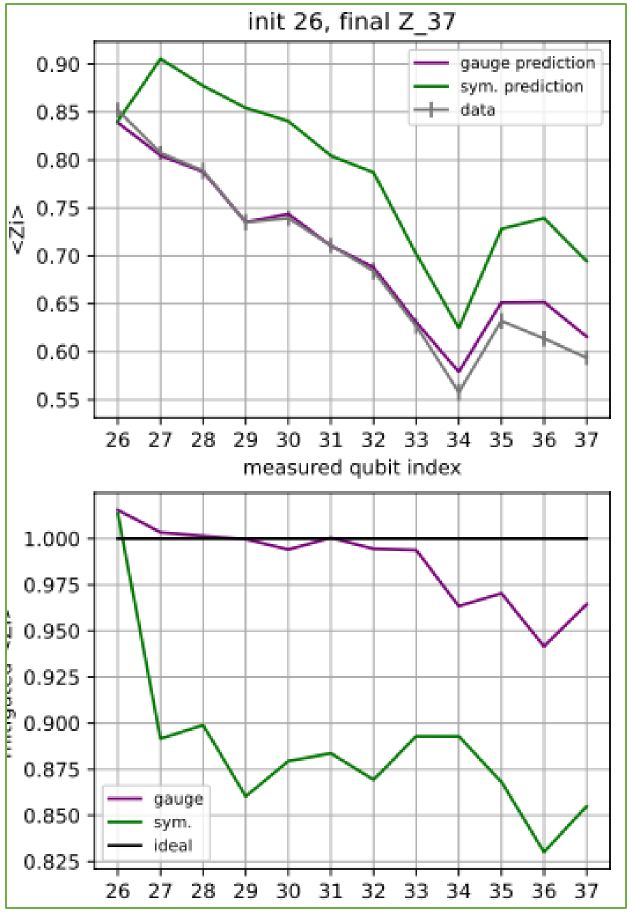


Manuscript in preparation, 2025.

Ring Experiments: 92Q results



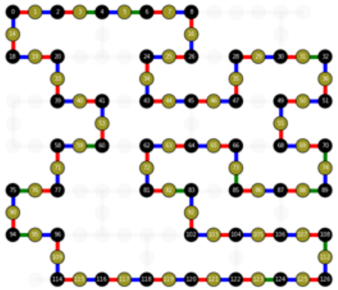




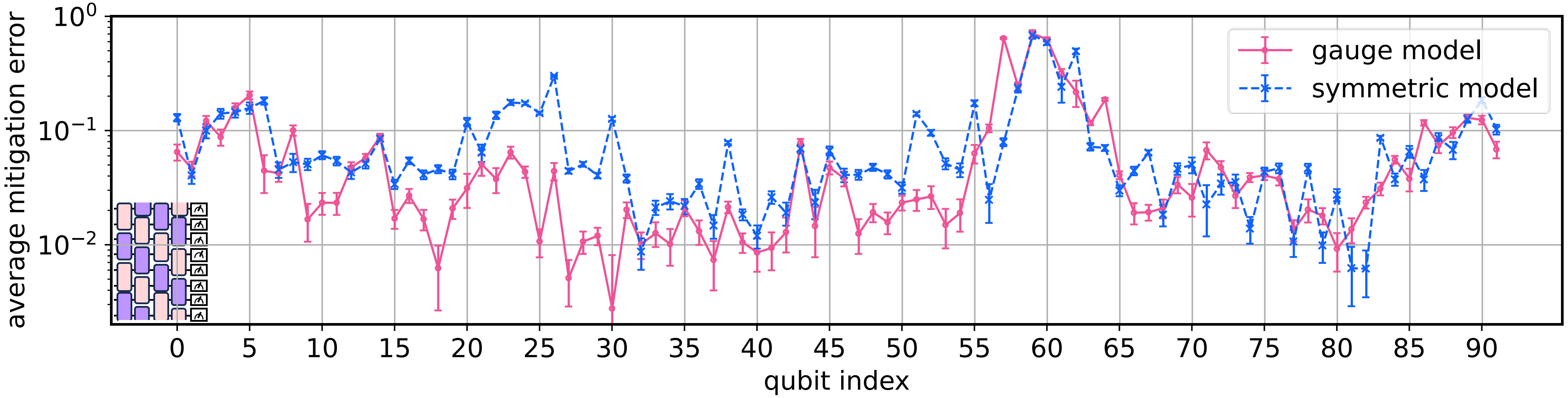
Target: Certain Pauli Path from weight-1 Pauli to weight 1-Pauli

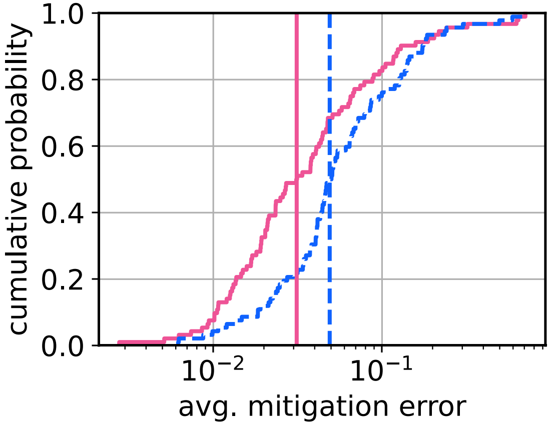
Manuscript in preparation, 2025.

Ring Experiments: 92Q results



12 steps





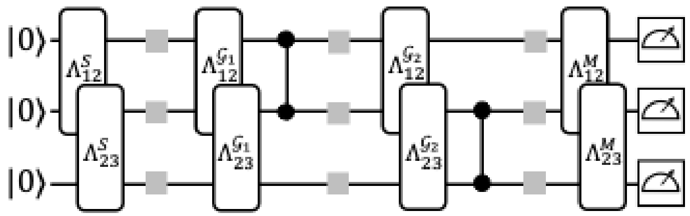
4.9% down to 3.1%

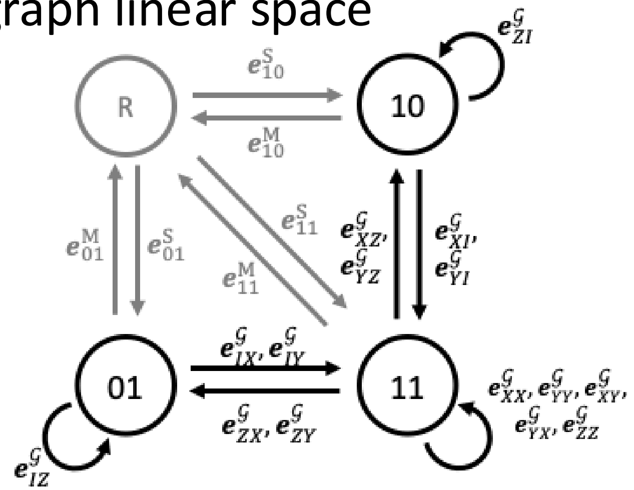
Manuscript in preparation, 2025.

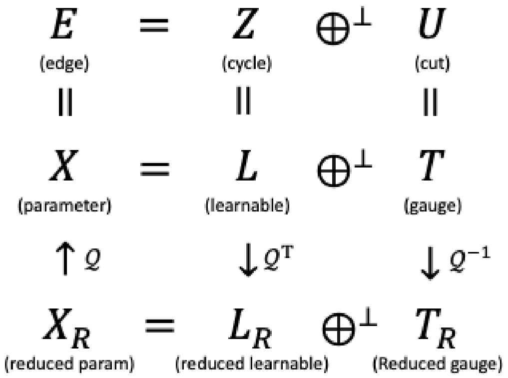
Summary

- We develop a framework of efficient self-consistent gate set Pauli noise learning

1. Characterization of learnable/gauge space via graph linear space
2. Case studies for local/quasi-local noise model



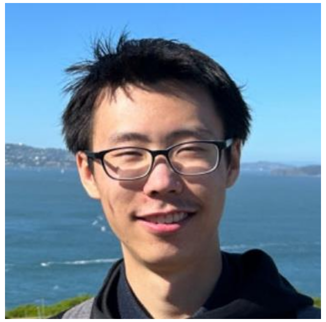




- Outlook:

1. Graph-theoretical techniques beyond Pauli noise model
2. Including MCMs *[Zhang et al. / Hines et al. PRXQ 2025]* and extending to logical learning.
3. Efficient self-consistent quantum error mitigation (To appear soon)
4. Optimal/Generic experiment design, fine-grained complexity analysis *[Hockings et al. PRXQ 2025]*

Thank you!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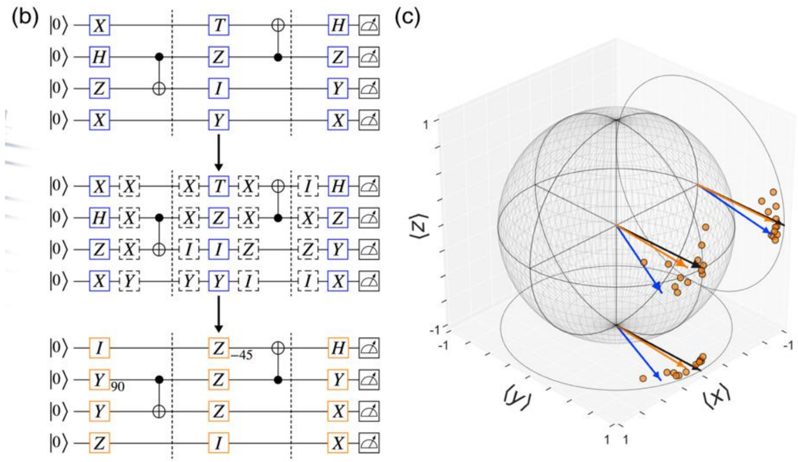
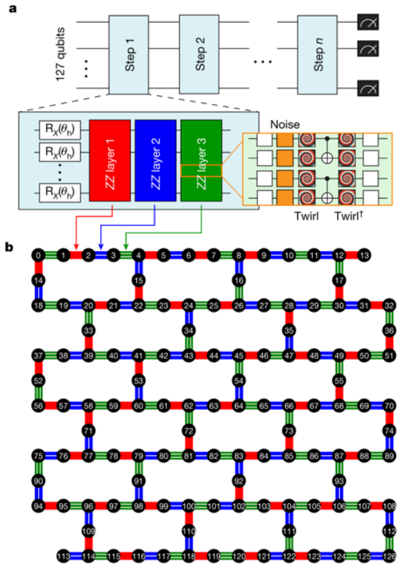
Appendix

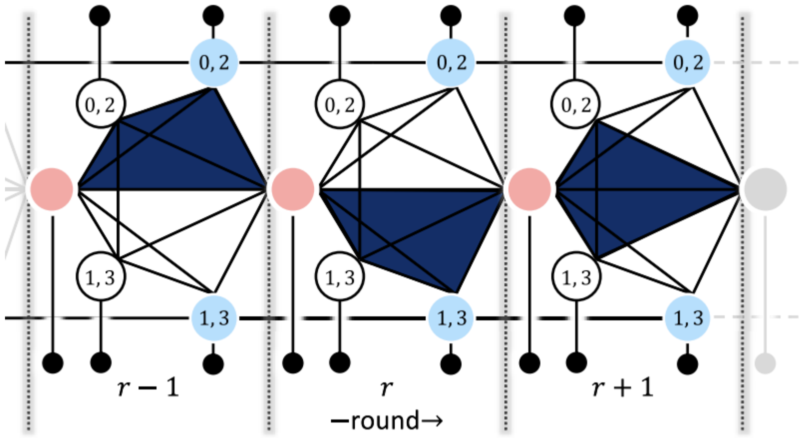
Why Pauli Noise Model



[Wallman and Emerson PRA 2016]
[A. Hashim et al., PRX 2021]



[Y. Kim et al., IBM, Nature 2023]



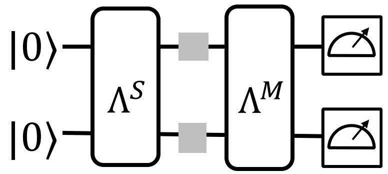
[E Chen et al., IBM, PRL. 2022]

- Generic noise can be twirled into Pauli channel via **randomized compiling**, given sufficiently good 1q gates.
- State-of-the-art **quantum error mitigation** techniques based on Pauli noise model
- Knowledge of Pauli noise rates useful for decoder optimizer in **QEC**.

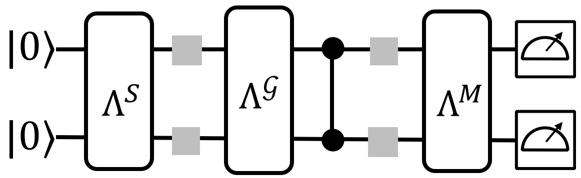
Prerequisite: Learn the Pauli noise model

Design learning circuits

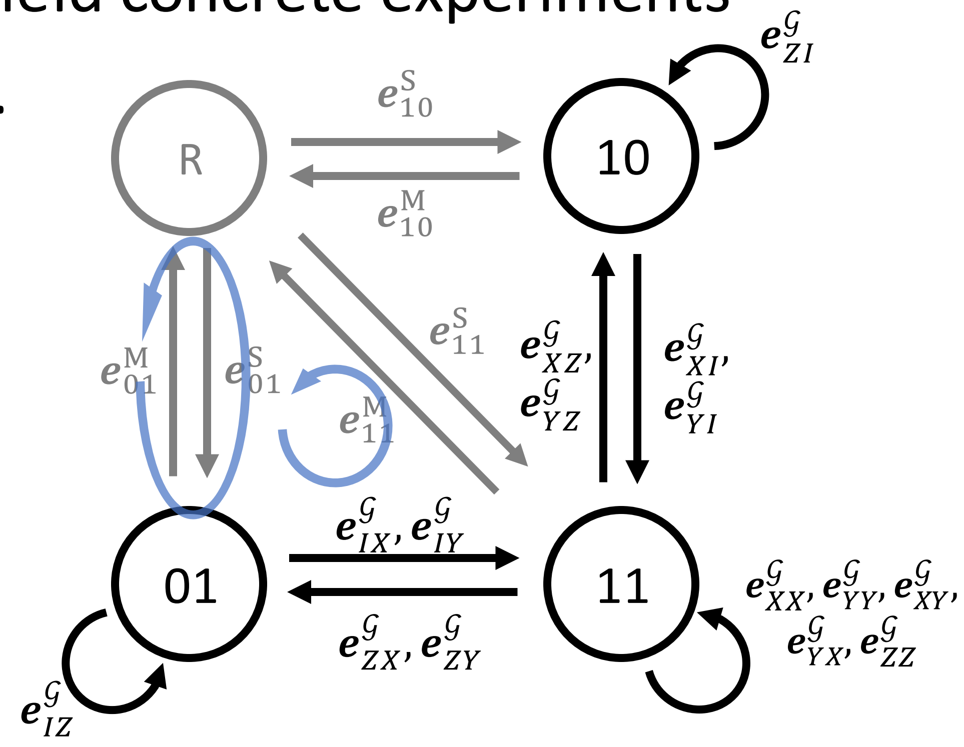
- Learnable space = Cycle space; Rooted cycles yield concrete experiments
- Find a **rooted cycle basis** and learn all of them.
- $\{\boldsymbol{e}_t^S + \boldsymbol{e}_t^M\} \cup \{\boldsymbol{e}^S_{\mathrm{pt}(a)} + \boldsymbol{e}^{\mathcal{G}}_a + \boldsymbol{e}^M_{\mathrm{pt}(\mathcal{G}(a))}\}$



Depth-0



Depth-1

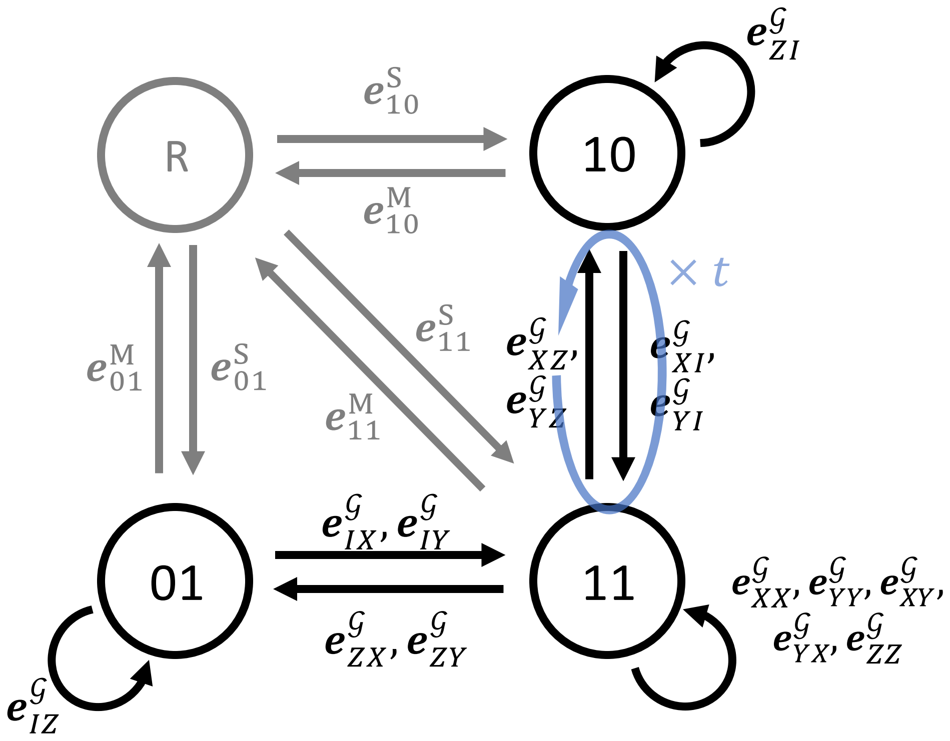


  - Learn $\mathcal{G} \in \mathfrak{G}$ one-by-one. No concatenation needed.
  - Only gives additive precision estimation

Learning to relative precision

- One often hopes to learn noise parameters to relative precision
  - With a small number of measurements
- Key: **Amplify** certain noise parameters using **concatenation** $\lambda^S_{\mathrm{pt}(a_1)}(\lambda^{\mathcal{G}_1}_{a_1}\lambda^{\mathcal{G}_2}_{a_2}\cdots\lambda^{\mathcal{G}_M}_{a_M})^t\,\lambda^M_{\mathrm{pt}(a_M)}$
  $= \exp(-(\boldsymbol{x}_{SPAM} + \boldsymbol{x}^t_{\mathrm{cycles}})$
- Theorem (informally):

  Any cycle consisting only of gate noise params can by amplified and learned via concatenation



- Cycles (of gate noise params) ≈ Germs or Tuples in GST