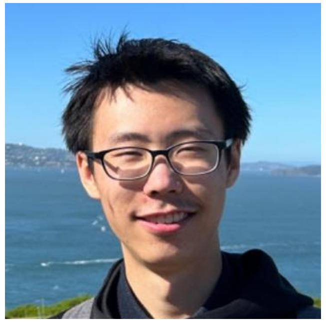
Generalized Cycle Benchmarking Algorithm for Characterizing Mid-Circuit Measurements

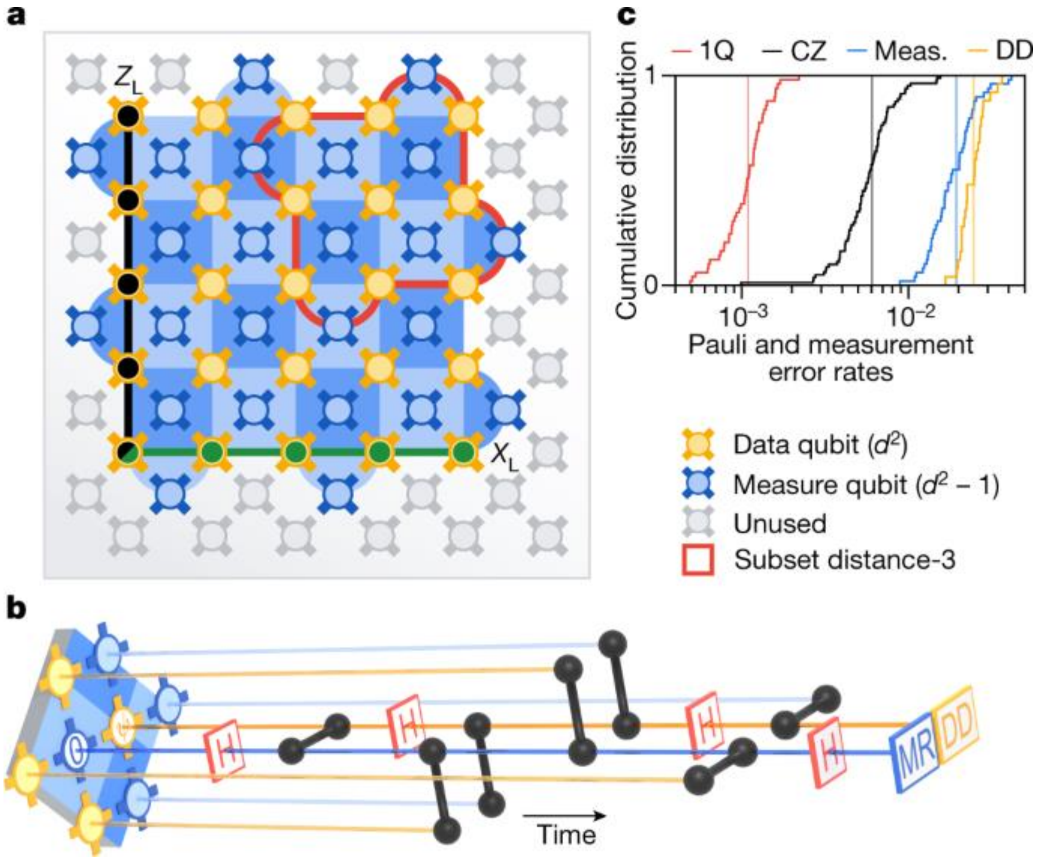
Senrui Chen (U of Chicago)

APS March Meeting 2025

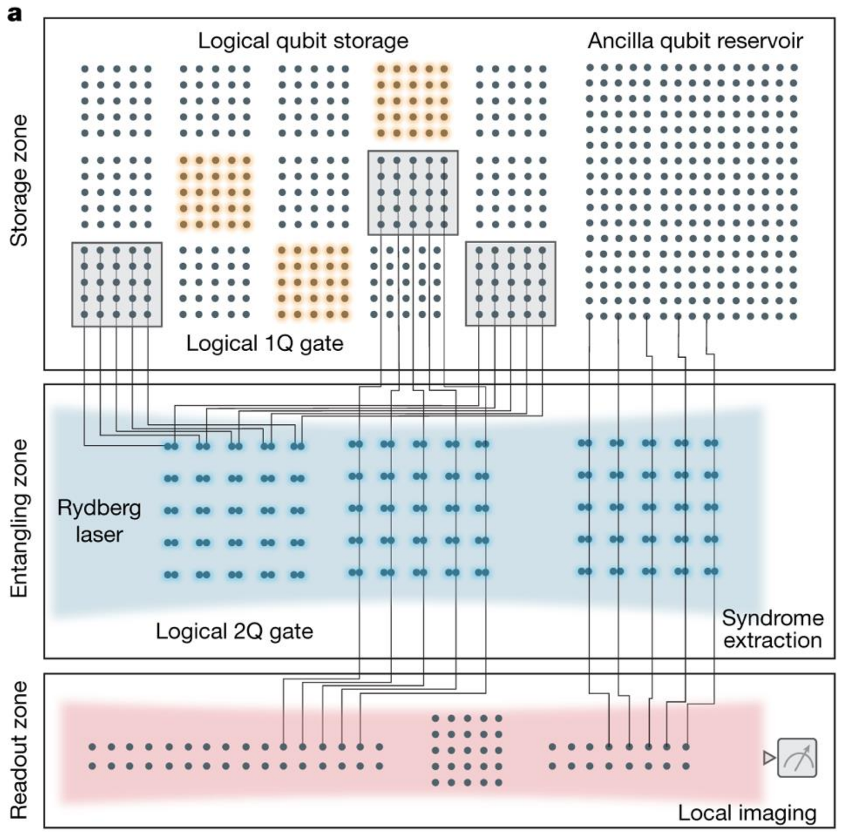


Based on [Zhihan Zhang, SC, Yunchao Liu, Liang Jiang. PRX Quantum **6**, 010310 (2025)]

Mid-Circuit Measurements (MCMs)



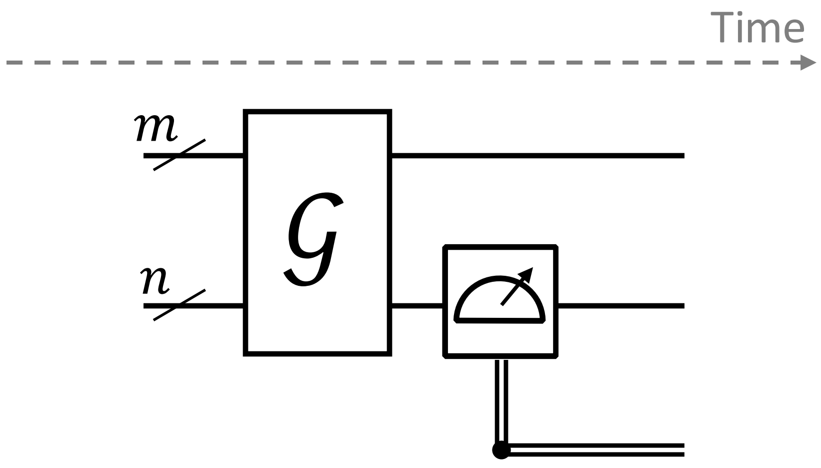
[Google Quantum AI, Nature (2023)]



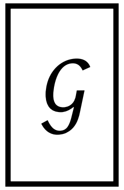
[D. Bluvstein et al., Nature (2024)]

- [Quantinuum, PRX 11, 041058 (2021)], [K. Singh et al., Science 380 (2023)], …

Mid-Circuit Measurements (MCMs)

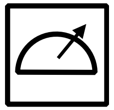


Ideal MCM



Clifford gates

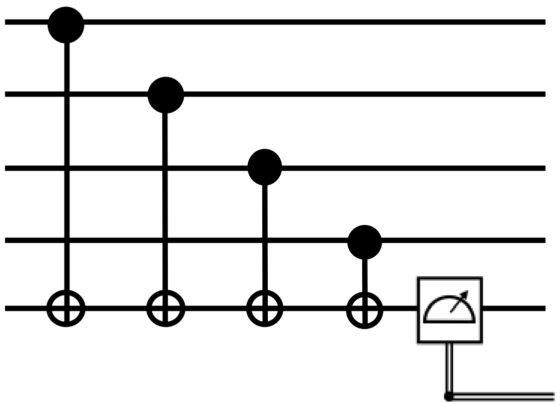
Quantum Wires

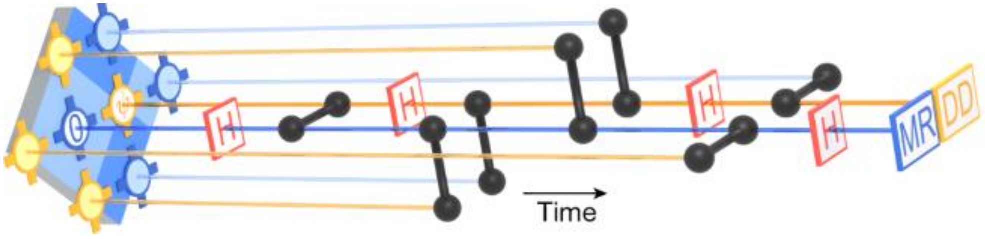


Computational basis measurements

Classical Wires

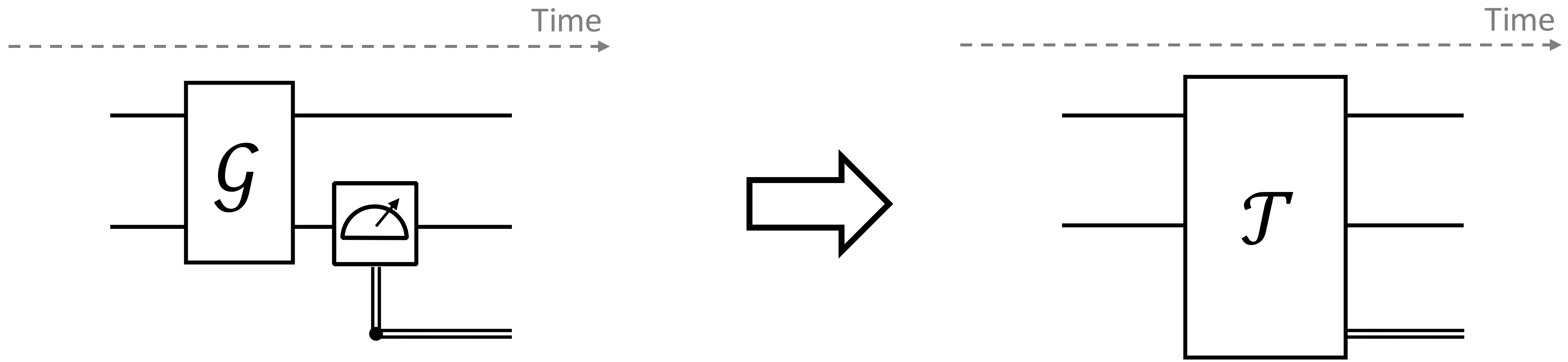
- E.g. Syndrome Extraction Circuits





[Google Quantum AI, Nature (2023)]

MCMs with Noise



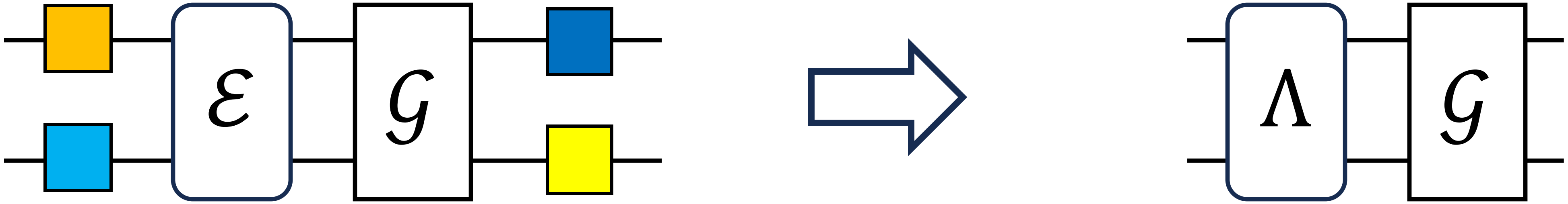
Ideal MCM

Noisy MCM

- **Quantum Instruments** (i.e. Q->QC channels). $\mathcal{T}(\rho) = \sum_j \mathcal{E}_j(\rho) \otimes |j\rangle\langle j|$
  - Where $\mathcal{E}_j$'s are completely-positive channels such that $\tilde{\mathcal{T}}$ is trace-presevering
- How to characterize a noisy MCM?
  - Knowledge of noise enables benchmarking, error mitigation, improved QEC, …
  - Challenge: general quantum instruments are complicated

Twirling for Clifford gates

- Let’s look at noise characterization for multi-qubit Clifford gate
- **Randomized Compiling**[1] tailors Clifford gate-dependent noise into Pauli channel
  - Require: Good single-qubit gates



Color box: Randomized Pauli Gates
(Sampled in a designed way)

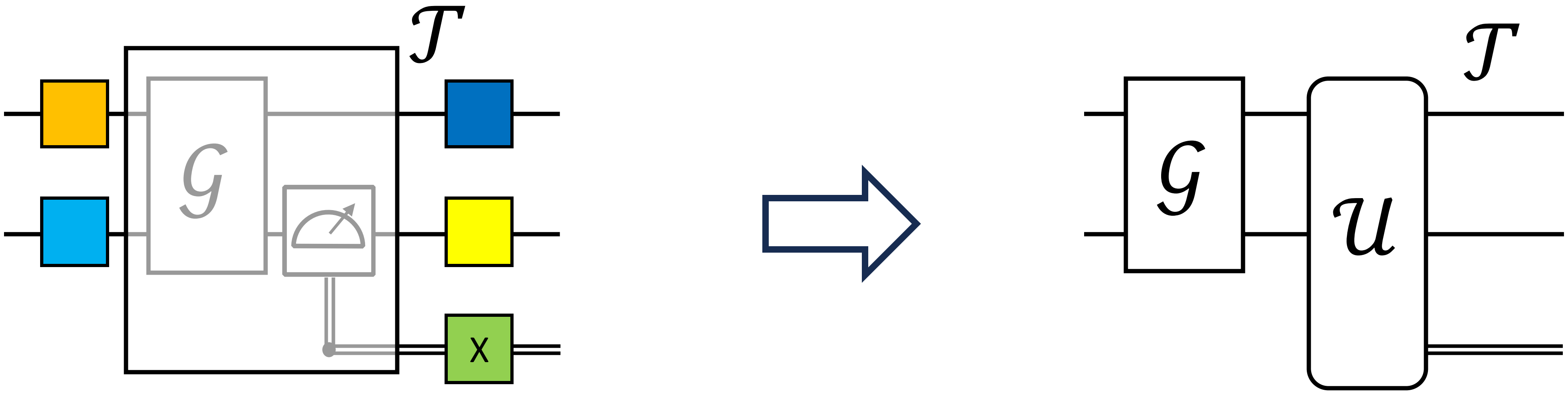
Pauli channel: $\Lambda(\rho) = \sum_{P \in \{I,X,Y,Z\}^n} p_P \, P \rho P$

- Protocols such as Cycle Benchmarking (CB)[2] can then be used to characterize Pauli noise channel SPAM-robustly.
  - I.e., Robust against state-preparation-and-measurement (SPAM) error.

[1] Joel Wallman and Joseph Emerson, PRA 2016. *[2] Erhard et al. Nat. Comm. 2019.*

Twirling for MCMs

- Fortunately, Randomized Compiling generalized to MCMs [2]



Color box: Randomized Pauli Gates
(Sampled in a designed way)

Uniform Stochastic Instrument

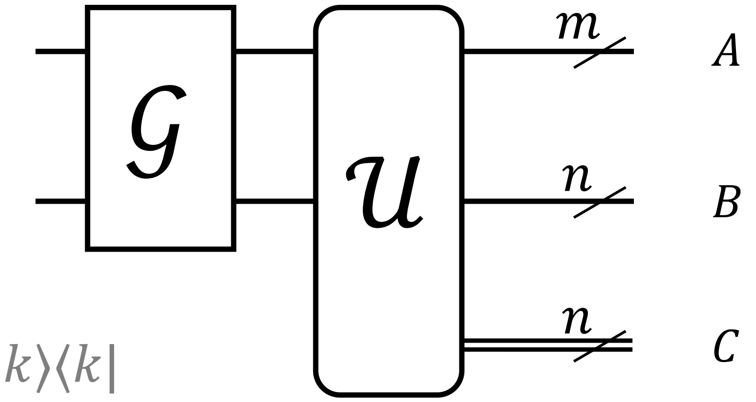
- The twirled quantum instrument becomes $\mathcal{T} = \mathcal{U} \circ \mathcal{G}$
- Where $\mathcal{U}$ is called a **Uniform Stochastic Instrument** (next slide)

[2] Stefanie Beale and Joel Wallman, arXiv: 2304.06599 (2023).

Uniform Stochastic instrument

- In the Pauli-Liouville representation:

$$\mathcal{U} = \sum_{\substack{P \in \{I,X,Y,Z\}^m, \\ k,a,b \in \{0,1\}^n}} p_{a,b}^{P}\, \mathcal{P}^A \otimes |k+a+b)(k|^B \otimes |k+a)^C$$



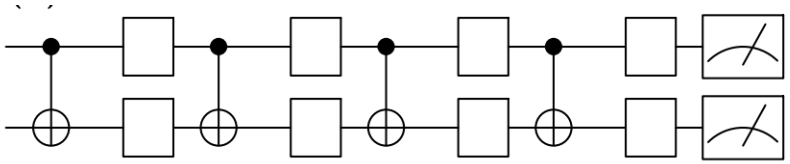
  - $\mathcal{P}(\rho) = P\rho P$, $|k)$ is vectorization of computational basis state $|k\rangle\langle k|$

- Intuitively, $\mathcal{U}$ characterized by a joint prob. dist. $p_{a,b}^{P}$ as follows
  1. Input $|k\rangle^B$, measurement outcome is wrongly recorded as $|k+a\rangle^C$
  2. Post-measurement state is contaminated as $|k+a+b\rangle^C$
  3. A Pauli error P^A happens at system A as back action.
- The goal is now to characterize $p_{a,b}^{P}$ (the error rates).

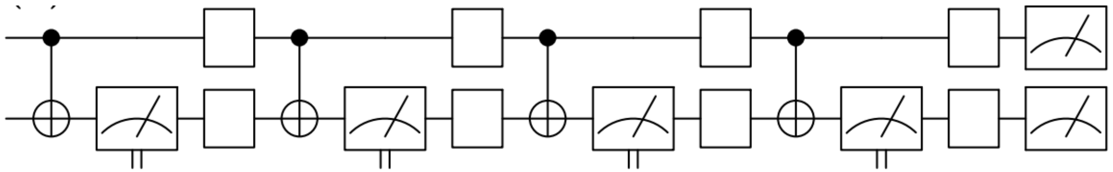
[2] *Stefanie Beale and Joel Wallman, arXiv: 2304.06599 (2023).*

Our Contributions

- We proposed a Generalized Cycle Benchmarking algorithm to characterize MCMs
  - The algorithm is robust against SPAM noise (same as RB and CB).

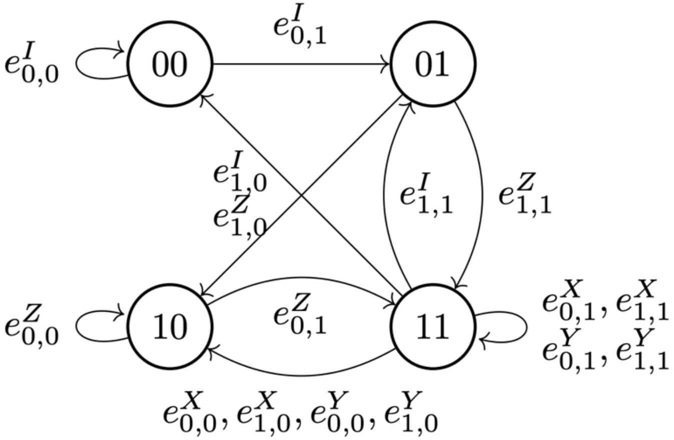


Standard Cycle Benchmarking



Generalized CB for MCMs

- Conversely, we develop a theory on the “learnability” of noisy MCMs
  - We give a graph-theoretic characterization of what parameters are (SPAM-robustly) learnable and which are gauge params.
  - Generalizing Learnability of Pauli noise [SC, Y Liu et al. NC 2023]
  - **Thm**. Generalized CB can extract every parameter that is learnable of a noisy MCM.



*[Zhihan Zhang, SC, Yunchao Liu, Liang Jiang. PRX Quantum **6**, 010310 (2025)]*

Generalized CB Alg.

- **Key ingredient**: Fourier Transform of the error rates $p_{a,b}^{P}$

$$\lambda_{x,y}^{Q} = \sum_{a,b\in\{0,1\}^n,\ P\in\mathbb{P}^m} (-1)^{a\cdot x+b\cdot y+\langle P,Q\rangle}\ p_{a,b}^{P}\ .$$

  - $\langle P,Q\rangle = 0$ if P commutes with Q. $\langle P,Q\rangle = 0$ elsewise.
  - This generalizes Walsh-Hadamard transformation between Pauli error rates / fidelities
  - We refer to $\lambda_{x,y}^{Q}$ as (generalized) Pauli fidelities.
- Twirled MCM $\mathcal{T} = \mathcal{U} \circ \mathcal{G}$ where the USI can be expressed as

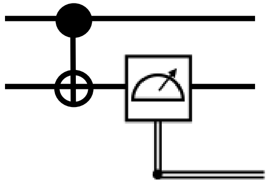
$$\mathcal{U} = \frac{1}{2^{2n+m}} \sum_{x,y,Q} (-1)^{k\cdot(x+y)} \lambda_{x,y}^{Q} |Q, Z^y)(Q, Z^x|$$

  - Z^x is an n-qubit Pauli, with ith entry being Z if $x_i = 1$ and I if $x_i = 0$.
  - $\lambda_{x,y}^{Q}$ are like the "transition amplitude" from Pauli (Q, Z^x) to Pauli (Q, Z^y)
    - One (Q, Z^x) can transfer to multiple (Q, Z^y)! Different from Pauli channel, NOT eigenvalues.

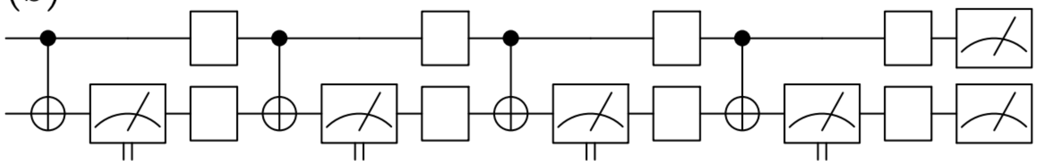
Generalized CB Alg.

$$\mathcal{T} = \frac{1}{2^{2n+m}} \sum_{x,y,Q} (-1)^{k\cdot(x+y)} \lambda^{Q}_{x,y} |Q, Z^{y})(Q, Z^{x}|\mathcal{G}$$

- **Goal**: Learn functions of $\lambda^{Q}_{x,y}$ as **Pauli fidelities path** (Monomial of λ's)
- Repeat MCMs and measure certain Pauli ops, yields sum of different **paths.**
- Use mid-circuit measurement outcomes + Fourier transform to single out one **path**.
- **Cancel SPAM noise** by differing two paths with same start/end Pauli ops.



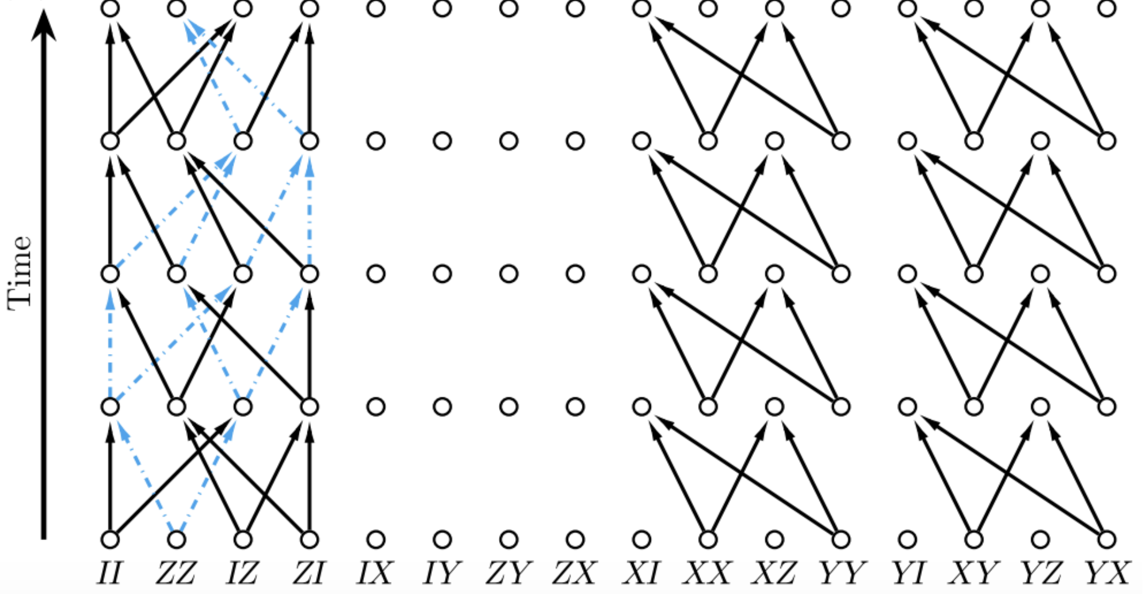
(b)



1. Prepare Pauli eigenstate
2. Apply repeated MCMs
3. Estimate Pauli observable
4. *Perform Fourier transform on measurement outcome*
5. Obtain sum of log Pauli fidelities
6. Perform Fourier transform, obtain Pauli error

Our protocol

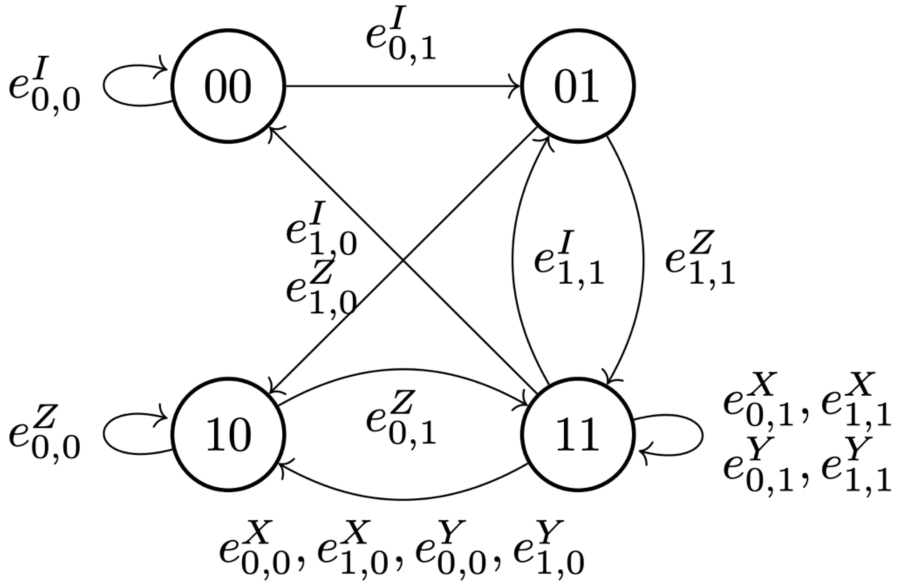
(b)

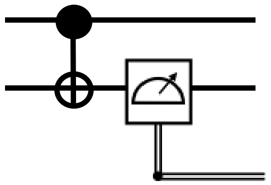


Learnability of noisy MCMs

$$\mathcal{T} = \frac{1}{2^{2n+m}} \sum_{x,y,Q} (-1)^{k\cdot(x+y)} \lambda^{Q}_{x,y} |Q, Z^{y})(Q, Z^{x}|\mathcal{G}$$

- So, what functions of $\lambda^{Q}_{x,y}$ are learnable?
- Encode the transition of Paulis in a graph
  - Only record the “support” or “pattern” of Pauli since we assume 1q gate to be noiseless
- **Thm (informal).** The space of learnable functions on $\{\lambda^{Q}_{x,y}\}$ equals to the space of Cycles on the pattern transfer graph.

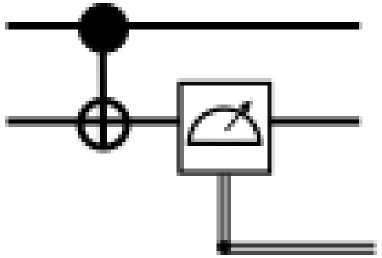


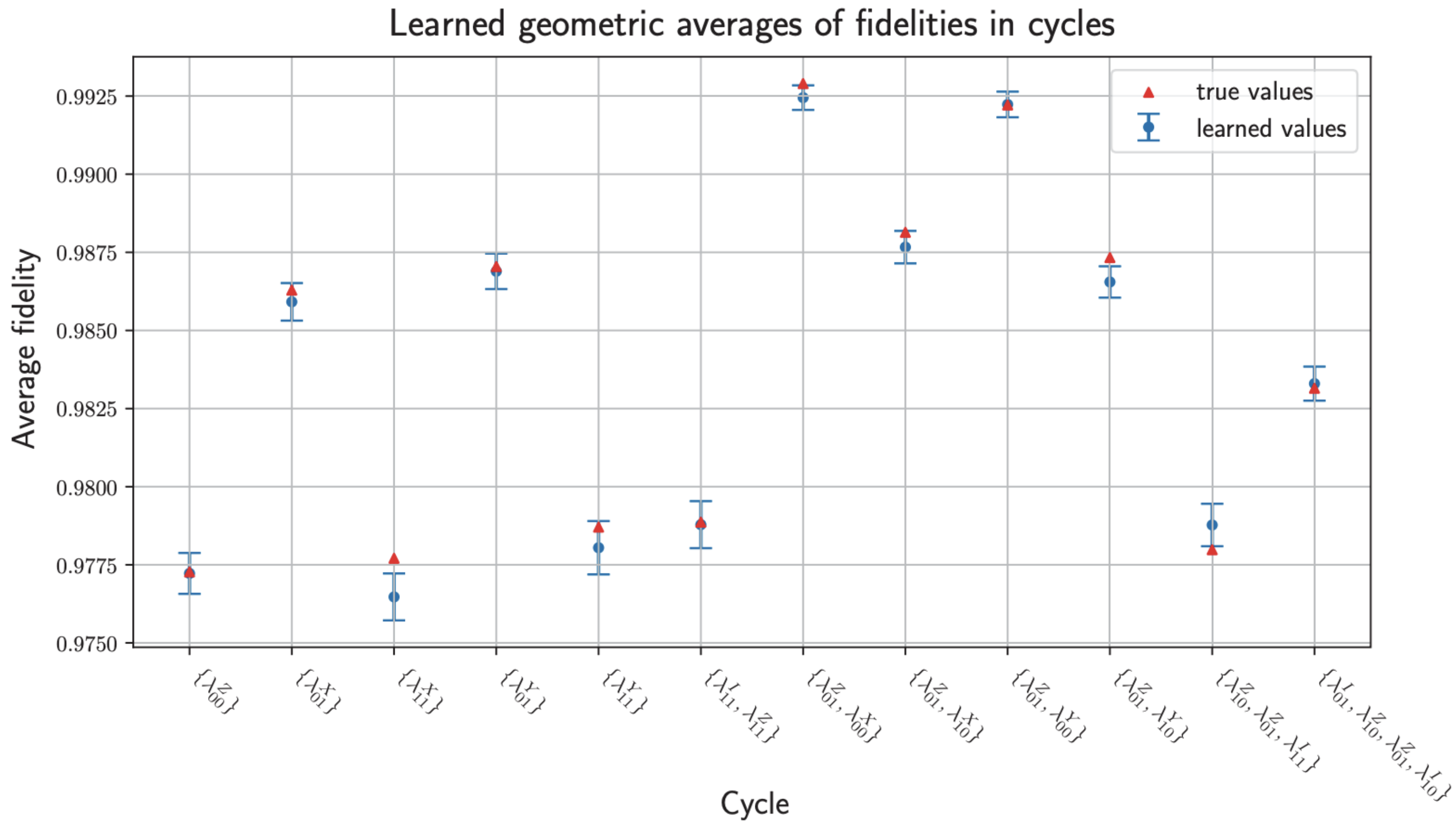
Pattern Transfer Graph for 

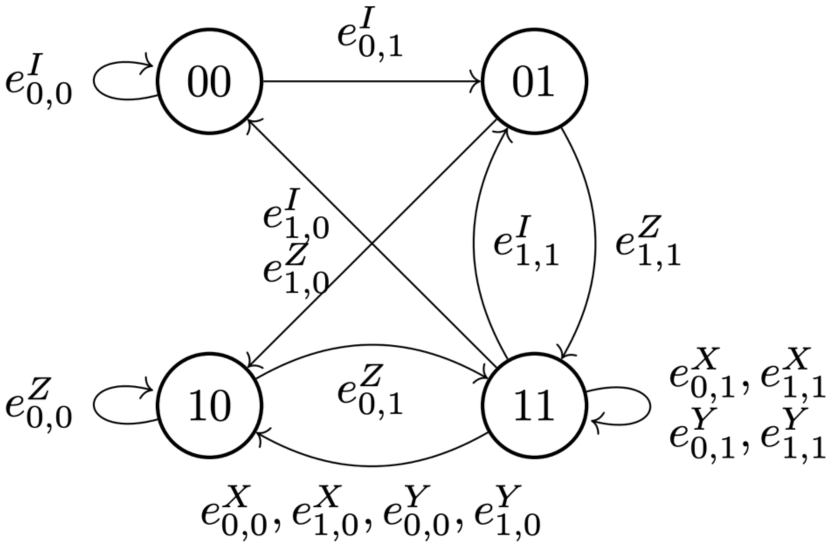
Proof Sketch: **Cycles** can be learned via generalized CB;
Cuts can be shown to be gauge transform;
Cycles $\oplus$ **Cuts** gives all noise params.

Numerics

- Generalized CB can indeed learn all 13 learnable DOFs for →



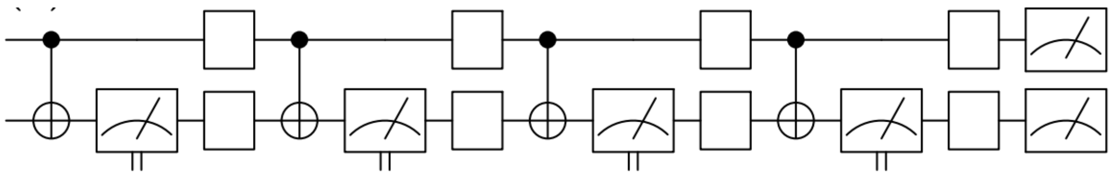




Summary and Outlook

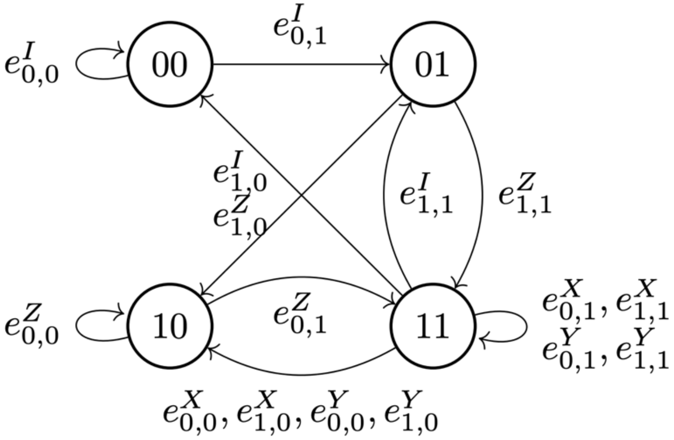
Results:

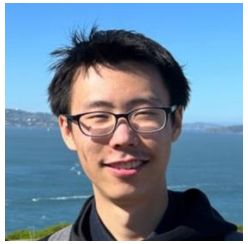
- A Generalized CB algorithm to characterize MCMs
- Learnability of noisy MCMs



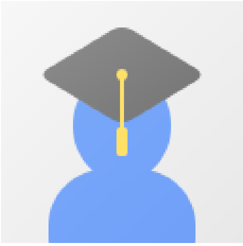
Outlook:

- Scalability: Locality constraints, complexity analysis
- Applications to Quantum Error Mitigation
- Beyond Uniform Stochastic Instrument
- Experimental implementation





Zhihan Zhang



Yunchao Liu

Liang Jiang